Institutio Mathematica

26

OR, A

MATHEMATICAL INSTITUTION.

Shewing the Construction and Use of the Natural and Artificial *Sines*, *Tangents*, and *Secants*, in *Decimal* Numbers, and also of the Table of *Logarithms*.

In the General solution of any Triangle, whether *Plain* or *Spherical*.

WITH

Their more particular application in

ASTRONOMIE,
DIALLING, and
NAVIGATION.

By JOHN NEWTON.

LONDON,
Printed for *William Fisher* at the *Postern-gate* near Great *Tower-hill*, 1671.

TO THE

COURTEOUS Reader.

ALthough *Mathematicall studies have for these many years been much neglected, if not contemned, yet have there been so many rare inventions found, even by men of our own Nation, that nothing now seems almost possible to be added more: as in other studies so may we say in these,* nil dictum quod non dictum prius. *We at the least must needs acknowledge that in this we have presented thee with nothing new, nothing that is our own,* Ex integra Græca integram Comœdiam, hodie sum acturus, Heautontimorumenon, *saith* Terence, *thus famous Comedian: translation was his apologie, transcription and collection ours: this* only

this we have endeavoured, that the first principles and foundations of these studies (which until now were not to be known, but by being acquainted with many Books.) might in a due method, and a perspicuous manner, be as it were at once, presented to thy view, and serve as a perfect INSTITUTION MATHEMATICAL, unto such as have as yet learned nothing but Arithmetick.

To that purpose we have first laid down such propositions Geometricall, out of Euclide, Pitiscus, and others, as must be known to such as would understand the nature and mensuration of all Triangles. Next we have proceeded to the affections of Triangles in the generall, and thence to the composition of the Sines, Tangents, and Secants Naturall, in which we have for the most part followed the Rules prescribed by Pitiscus, in some things we have taken the direction of Snellius, and in the trisection and quinquisection of an angle we have proceeded Algebraically, with those two famous Mathematicians of our age and Nation, Brigges, and Oughtred; and because the Algebraicall work is of it selfe abstruse and intricate, to those that are not acquainted with it, we have insisted the more upon it, and by our explanation we have endeavoured to make it plain and easie; and

that

that nothing may be wanting, which either former ages or our own (by Gods blessing upon their industry) have afforded to us, we have to the composition of the Natural Canon, added out of Briggs and Wingate the construction of the Logarithms of any numbers, and consequently how to make the Logarithms of the Naturall Sines, Tangents, and Secants. This done, the proportions in the usuall Cases of all Triangles both Plain and Sphericall, we have first cleared by Demonstration out of Pitiscus, Gelibrand, Norwood, and others, and then explained the manner of the work in Natural and Artificial Numbers both, and so conclude the first Part of our Institution.

And in the second Part we have made our application of all the former unto Astronomie first, and then to Dialling and Navigation. In our application to Astronomie, we have furnished you with a Table of the Suns Motion, whereby to calculate his place in the Zodiac in Decimal numbers, and without which most of the other Problemes would be found (if not uselesse) yet very intricate and obscure, that being, for the most part, one of the three terms supposed to be given in Astronomical computations.

In the Chapter of Dialling, you have the Sphears projection, according to the direction

[illegible] Wels, in his Art of Shadows, and how to draw the houre-lines of all the severall Dials which he hath contrived from thence, we have briefly shewed; and in the finding of all the arches in these cases necessary, we have kept our selves to our own CANON, which doth exhibit the degrees of the Quadrant in Centecimal parts or minutes.

In the Chapter *of* Navigation, *you have first the division of the* Sea-mans Compasse, *next the description and making of the* Sea-Chart, *as* Edward Wright *our worthy Countrey-man hath given us the* Demonstration *thereof in his* Book *entituled,* The correction of Errours in Navigation: *to these we have added such other* Problemes *as are now amongst our* Sea-men *of most frequent use; annexing thereunto a* Table *of* meridionall parts, *and other* Tables *usefull as well in* Dialling *as in* Navigation, *and all these in* Decimall numbers, *it being indeed our aim (as much as in us lieth) not only to promote these studies by this our* Compendium *of the first rudiments of* Mathematical learning, *as in relation to the matter therein to be considered, but by such expeditious and advantagious wayes of working also, as have been lately found, or former ages have commended to us; amongst which there is none more excellent*

lent then that which is performed by Decimal numbers; fully to explicate the manner and worth whereof were matter enough for a whole Treatise, and therefore not to be expected in a short Epistle: It would indeed be very impertinent to intermeddle any further with it here, then in our Institution it selfe is already explained, in which thou maist perceive Addition and Subtraction of Degrees, Minutes, and Seconds, to be performed as in Vulgar numbers, without any Reduction to their severall Denominations, Multiplication is performed by the addition of Cyphers, and Division by the cutting off of Figures. Others that have either spent more time, or made a farther progresse in these ravishing Studies, might (if they would have taken the paines) have haply presented thee with more, and in a lesser room: The most of this was at the first collected for our private use, and now published for the good of others.

John Newton.

ERRATA.

Page	Line	
6	37	Read, or termes.
17		For B 9 A, r. B 9 C.
16	20	read, 3 times 6 is 18.
22	12	r. one third
23	25	r. one third
30	19	r. triangles in the following figure.
32	22	r. by the 18th.
34	21	r. are equiangled.
38	24	r. the arch C G K.
44	25	r. as of the first.
47	9	r. of 90 degrees.
49	24	r. by the 18.
55	20	r. that of the first.
60	4	r. by 18.
		r. a Rad. ae — AO44
63	30	Rad.
77	16	r. 347.
77	21	r. 3913.
77	22	r. 106087.
91	3	r. 24986.
108	1	r. K P.
112	26	r. B D
156	6	r. B D.
	7	r. D F.
	8	r. B F

A MATHEMATICALL Institution.

CHAP. I.

Geometricall Definitions.

Of things *Mathematicall*, there be two principal Subjects, *Number* and *Magnitude*, and each of these hath his proper Science.

The Science of *Number* is *Arithmetick*, and the Science of *Magnitude* is commonly called *Geometry*, but may more properly be termed *Megethologia*, as comprehending all Magnitudes whatsoever, whereas *Geometry*, by the very Etymologie of the word, doth seeme to restraine this Science to Land measuring only.

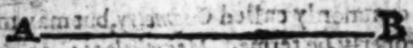
For this *Megethologia* or *Geometry*, [illegible]

ence of Magnitudes, we will ſet down ſuch grounds and principles as are neceſſary to be known, for the better underſtanding of that which follows, preſuming that the reader hereof hath already gotten ſome competent knowledge in *Arithmetick*.

Concerning then this Science of *Magnitude*, two things are to be conſidered: Firſt, the ſeverall heads to which all *Magnitudes* may be referred: And then ſecondly, the terms and limits of thoſe *Magnitudes*.

All *Magnitudes* are either *Lines*, *Plains*, or *Solids*, and do participate of Length, Breadth, or Thickneſſe.

1. *A Line* is a ſuppoſed length, or a thing extending it ſelf in length, without breadth or thickneſs, whether it be a right line or a crooked; and may be divided into parts in reſpect of his length, but admitteth no other diviſions, as the line A B.



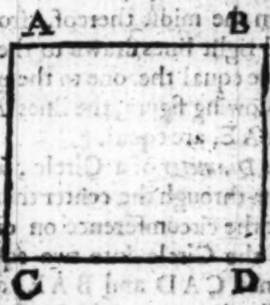
2. The ends or limits of a line are points, as having his beginning from a point, and ending in a point, and therefore a *Point*

hath

hath neither part nor quantity, it is only the term or end of quantity, as the points A and B are the ends of the aforesaid line A B, and no parts thereof.

3. A *Plain* or *Superficies* is the second kind of magnitude, to which belongeth two dimensions, length, and breadth, but not thickness.

4. As the ends, limits or bounds of a line are points confining the line, so are lines the limits, bounds and ends inclosing a Superficies; as in the figure you may see the plain or Superficies here inclosed with four lines, which are the extreams or limits thereof.



5. A *Body* or *Solid* is the third kinde of magnitude, and hath three dimensions be-

longing to it, length, breadth, and thickneſſe. And as a point is the limit or term of a line, and a line the limit or term of a Superficies, ſo likewiſe a Superficies is the end or limit of a Body or Solid, and repreſenteth to the eye the ſhape or figure thereof.

6. A *Figure* is that which is contained under one or many limits, Under one bound or limit is comprehended a Circle, and all other figures under many.

7. A *Circle* is a plain figure, contained under one round line, which is called a circumference, as in the Figure following, the Ring C B D E is called the circumference of that Circle.

8. The *Center* of a Circle is that point which is in the midſt thereof, from which point, all right lines drawn to the circumference are equal the one to the other; as in the following figure, the lines A B, A C, A D, and A E, are equal.

9. The *Diameter* of a Circle, is a right line drawn through the center thereof, and ending at the circumference on either ſide, dividing the Circle into two equal parts, as the lines C A D and B A E, are either of them the diameter of the Circle B C D E, becauſe that either of them doth paſſe through the center A, and divideth the whole

whole Circle into two equall parts.

10. The *Semidiameter* of a Circle is halfe the Diameter, and is contained betweene the center and one side of the Circle; as the lines A B, A C, A D, and A E, are each of them the Semidiameters of the Circle C B D E.

11. A *Semicircle* is the one halfe of a Circle drawn upon his Diameter, and is contained by the halfe circumference and the Diameter; as the Semicircle C B D is halfe the Circle C B D E, and contained betwixt the Diameter C A D.

likewise into two equall parts, and or A B C, the *Quadrant* or fourth part of the Circle.

13. A *Segment* or portion of a Circle is a figure contained under a right line and a part of the circumference of a Circle, either greater or lesser than the Semicircle; as in the former figure, F B G H is a segment or part of the Circle C B D E, contained under the right line F G, lesser than the Diameter C A D.

14. By the application of severall lines in termes of a Superficies one to another, are made Parallels, Angles, and many sided Figures.

15. A *Parallel* line is a line drawn by the side of another line, in such sort that they may-

12. A *Quadrant* is the fourth part of a Circle, and is contained betwixt the Semidiameter of the Circle, and a line drawn perpendicular unto the Diameter of the same Circle, from the center thereof, dividing the Semicircle into two equal parts, of the which parts the one is the Quadrant or fourth part of the same Circle. Thus, the Diameter of the Circle B D E C is the line C A D, dividing the Circle into two equal parts, then from the center A raise the perpendicular A B, dividing the Semicircle likewise into two equal parts, so is A B D, or A B C, the Quadrant or fourth part of the Circle.

13. A *Segment* or portion of a Circle is a figure contained under a right line and a part of the circumference of a Circle, either greater or lesser than the Semicircle; as in the former figure, F B G H is a segment or part of the Circle C B D E, contained under the right line F H G lesse than the Diameter C A D.

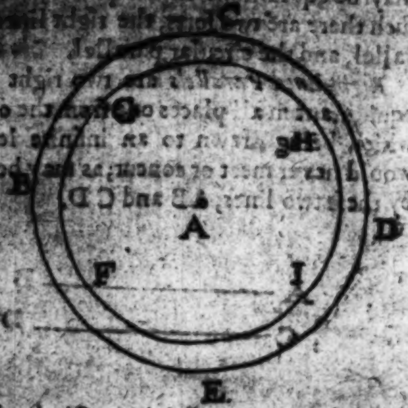
14. By the application of several lines in terms of a Superficies one to another, are made Parallels, Angles, and many sided Figures.

15. A *Parallel* line is a line drawn by the side of another line, in such sort that they may

may be equidistant in all places, and of such there are two sorts, the right lined parallel, and the circular parallel.

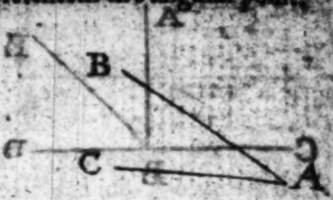
Right lined Parallels are two right lines equidistant in all places one from the other, which being drawn to an infinite length would never meet or concur; as may be seen by these two lines, A B and C D.

A *Circular Parallel* is a Circle drawn within or without another Circle, upon the same center, as you may plainly see by the two Circles B C D E, and F G H I, these Circles are both of them drawn upon the same center A, and therefore are parallel one to the other.

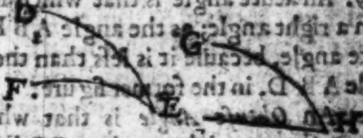


16. An *Angle* is the meeting of two lines in any sort, so as they both make not one line; as the two lines A B and A C incline one to the other, and touch one another in the point A, in which point is made the angle B A C. And if the lines which contain the angle be right lines, then it is called a right lined angle; as the angle B A C. A crooked lined angle is that which is contained of crooked lines; as the angle D E F: and a mixt angle is that which is contained both of a right and crooked line; as the angle G H I: where note that an angle is (for the

the two (right lines) [illegible] which [illegible] eth the angular point, as in the angle BAC, A representeth the angular point.



19. An acute angle is that which is lesse than a right angle; as the angle A B E is an acute angle, because it is lesse than the right angle A B D. in the former figure.



20. An Obtuse angle is that which is greater than a right angle; C B E in the former figure is greater than the angle ABC an obtuse angle.

All Angles are called Right, Acute, or Obtuse.

21. When a right line standeth upon a right line, making the angles on either side equal, either of those angles is a right angle, and the right line which standeth erect, is a perpendicular line to that upon which it standeth: As the line A B (in the following Figure) falling upon the line C B D perpendicularly, doth make the angles on both sides equal, that is the

A B C is equal to the angle A B D, and either of those angles is therefore a right angle.

19. An acute angle is that which is lesse than a right angle; as the angle A B E is an acute angle, because it is less than the right angle A B D, in the former figure.

20. An *Obtuse Angle* is that which is greater than a right angle; C B E in the former figure is greater than the angle ABC by the angle A B E, and therefore it is an obtuse angle.

21. The measure of every angle is the arch of a Circle described on the angular point, as in the following figure, the arch C D is the measure of the right angle CED. The arch B C is the measure of the acute angle B E C. And the arch B C D is the measure of the obtuse angle B E D. But of their measure there can be no certain knowledge, unlesse the quantity of those arches be exprest in numbers.

22. Every

22. Every Circle therefore is supposed to be divided into 360 equall parts, called Degrees, and every Degree into 60 Minutes, every Minute into 60 Seconds, and so forward. This division of the Circle into 360 parts we shall retain, but every Degree we will suppose to be divided into 100 parts or Minutes, & every Minute into 100 Seconds: and thus all Calculations will be much easier, and no lesse certain.

23. A *Semicircle* is the halfe of a whole circle containing 180 degrees. A Quadrant or fourth part of a circle is 90 degrees. And thus the measure of the right angle C E D is the arch C D 90 degrees. The measure of the acute angle B E C is the arch B C 30 degrees. And the measure of the obtuse angle BED is the arch BD 120 degrees.

24. The complement of an angle to a Quadrant is so much as the angle wanteth of

of 90 degrees, as the complement of the angle A E B 60 degrees is the angle B E C 30 degrees; for 30 and 60 do make a Quadrant of 90 degrees.

31. The complement of an angle to a Semicircle is so much as the said angle wanteth of 180 degrees, as the complement of the angle B E D 120 degrees, is the angle A E B 60 degrees; for 60 and 120 do make 180 degrees.

32. Many sided figures are such as are made of three, four or more lines, though for distinction sake, those only are so called which are contained under five lines or more at the least.

33. Four sided figures are such as are contained under four lines or terms, and are of divers sorts.

1. There is the Quadrat or Square whose sides are equall and his angles right.

2. The Long Square whose angles are right, but the sides unequall.

3. The Rhombus or Diamond, having equall sides but not equall angles.

4. The Rhomboides having neither equall sides nor equall angles, although the opposite sides and angles equall.

All other figures of four sides are called Trapezia or Tables. The distinction whereof

as

as also of all figures whatsoever, dependeth upon the knowledge of three sided figures, or Triangles, of which in the Chapter following.

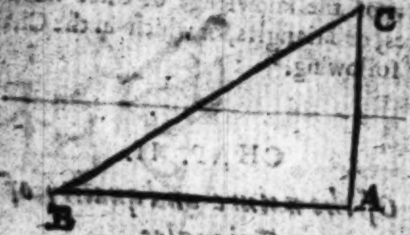
CHAP. II.

Of the nature and quality of Triangles.

1. A Triangle is a figure consisting of three sides and three angles.

2. Every of the two sides of any Triangle are the sides of the angle comprehended by them, the third side is the Base, as in the figure following, the sides A C and B C are sides of the angle A C B, and A B is the Base of the said angle.

3. Every side is said to subtend the angle that is opposite to that side; as the side A B subtendeth the angle A C B, the side A C subtendeth the angle A B C, and the side B C subtendeth the angle B A C: the greater sides subtend the greater angles, the lesser sides lesser angles, and equal sides equal angles.



4. Of Triangles there are diverse sorts; as,

1. There are Equilateral Triangles, having three equal sides.

2. There is an *Isoscheles*, which is a Triangle that hath two equal sides.

3. *Scalenum*, which is a Triangle whose sides are all unequal.

4. An *Orthigonium*, or a right angled Triangle, having one right angle.

5. An *Ambligonium*, or an obtuse angled Triangle, having in it one obtuse angle.

6. An *Oxigonium*, or an acute angled Triangle, having all his angles acute.

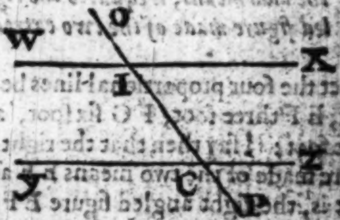
7. All these Triangles are either Plain or Spherical.

8. The sides of Plain Triangles in *Trigonometria* are right lines only, concerning which

which we have added these *Theorems* following.

9. Theorem. *If one right line cut through two parallel right lines, then are the angles opposite one against another equal.*

In the following Scheme the two lines WX and Y Z are parallel, and therefore the angles X I C, and I C Y are equal.



Demonstration.

The two angles X I C and W I C are equal to two right angles, as also I C Y and I C Z, because on the parallel lines at the points I and C there may be drawn two Semicircles, each of which are the measures of two right angles. If then the angle X I C be lesse than I C Y, the angle W I C must as much exceed the angle I C Z, and the angles X I C and Z C I would be lesse than

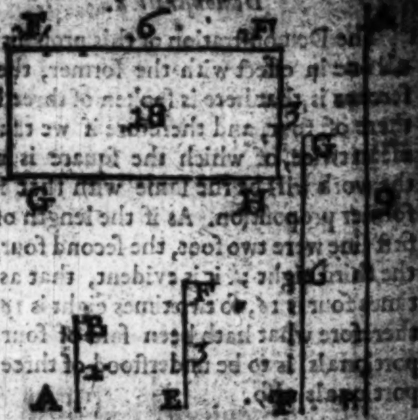
two

[illegible] and consequently the [illegible] W X and Y Z may be extended equally [illegible] X and Z til at length they shall concur [illegible] the lines W X and Y Z [illegible] and [illegible] angles [illegible] are equal.

[illegible] *Theorem, If four lines be [illegible] proportionals, the right angled figure made of the two means, is equal to the right angled figure made of the two extreams.*

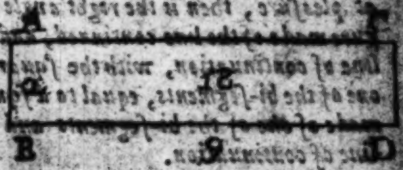
Let the four proportional lines be A B two [illegible], E F three foot, F G six foot, and B C [illegible] foot; I say then that the right angled figure made of the two means E F and F G, that is, the right angled figure E F G H, is [illegible] to the right angled figure made of the [illegible] A B and B C, that is, to the right angled figure A B C D, for [illegible] twice [illegible] 18, [illegible] three times is 18.

Demonstratio 8.



The Demonstration of this proposition is the same in effect with the former; the difference is, that here is spoken of three lines, there of four, and therefore if we take the middle line, of which the square is made, twice, it will be the same with that in the former proposition. As if the length of the first line were two foot, the second four, and the third eight; it is evident, that as four times four is 16, so two times eight is 16, and therefore what hath been said of four proportionals is to be understood of three proportionals also.

12. Theor. *If a right line being divided into two equal parts, shall be continued at pleasure, then is the right angle figure made of the line continued, and the line of continuation, with the square of one of the bi-segments, equal to a square made of the half bi-segment and the line of continuation.*



Demonstration.

The Demonstration of this proposition is all one in effect with the former, the difference is, that here is spoken of three lines, there of four, and therefore if we take the mean twice, of which the square is made, the work will be the same with that in the former proposition. As if the length of the first line were two foot, the second four, and the third eight; it is evident, that as four times four is 16, so two times eight is 16, and therefore what hath been said of four proportionals, is to be understood of three proportionals also.

12. Theor. *If a right line being divided into two equal parts, shall be continued at pleasure, then is the right angled figure made of the line continued, and the line of continuation, with the square of one of the bi-segments, equal to a square made of one of the bi-segments and the line of continuation.*

The line P Q is divided into two equal parts, the midst is C, to the same is added a right line, as Q N; and of the whole line P Q, and the added line Q N is made

P N as one line, and of this line P N, and the added line Q N is inclosed the right angled figure P M, and upon the halfe line C Q and the line of continuation Q N is made the square C F. Now if you draw the line Q G parallel to N F, and equal to the same, then is the right angled figure P M with the square of C Q that is, the square I G, equal to the square of C N, that is, the square C F.

Demonstration.

Forasmuch as C Q is equal unto M F, the which is also equal unto I O or I L, it followeth that I G is a Square, which with the right angled figure P M is equal to the square C F, because the right angled figure G M is equal to C O, which is also equal to P I.

To divide a right line in two parts so that the right angled figure contained of the whole line and one part, shall be equall to the square of the other part.

Let the right line given be A B, upon the line A B make a square, as A B C D, divide the side A D in two equall parts, call it M, from M draw a line to B, produce A D to H, so that M H be equall to M B, and upon A H make a square, A H G F. Then extend G F to E, and the right angled figure F C, being contained of the whole line F E (which is equall to A B) and the part B F, equall to the square of the other part A F, that is, to the square A H G F.

right

right angled figure [illegible]
and H A, in the right angled [illegible]
and H G, as the figure G [illegible]
square of A M, are together equall [illegible]
square of H M, being equall [illegible]
followeth, that if we take away the [illegible]
of A M, common to both, that the square
of A B, that is, the square [illegible]
to the right angled figure H G [illegible]
common right angled figure [illegible]
from them both, there shall remain the
right angled figure F G, equall to the square
A H F G, which was to be proved.

14. Theor. *To divide a right line given by extream and mean proportion.*

A right line is said to be divided by extream and mean proportion, when the whole is to the greater part, as the greater is to the lesser. And thus a right line is divided, as the right line [illegible] in the preceding Diagram, in the point [illegible] divided in extream and mean proportion, that is, As A B, is to A [illegible]

Demonstration.

Forasmuch as the right lined figure [illegible] cluded with A B and F B, [illegible]

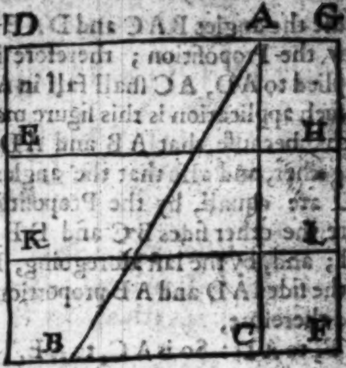
FBCE is equal to the ſquare of AF, that is, to the ſquare AFGH; it followeth, by the eleventh Theorem of this Chapter, that the line AB is divided in extream and mean proportion; that is, As AB, is to AF: So is AF, to FB.

15 Theor. *In all plain Triangles, a line drawn parallel to any of the ſides, cutteth the other two ſides proportionally.*

As in the plain Triangle ABC, KL being parallel to the baſe BC, it cutteth off from the ſide AC one fourth, and alſo it cutteth off from the ſide AB one third part: the reaſon is, becauſe the right line EH cutteth off one third part from the whole ſpace DGIB, & therefore it cutteth off one third part from all the lines that are drawn quite through that ſpace.

And hereupon parallel lines bounded with parallels are equal; as the parallels ED and GH being bounded with the parallels DG and NB are equal, for ſince the whole lines DB and GF are equall, DE and GH being one fourth part thereof, muſt needs be equal alſo.

16 Theor.



16 Theor. *Equiangled Triangles have their sides about the equall angles proportionall, and contrarily.*

Let AB C and A D E be two plain equiangled Triangles, so as the angles at B and D, at A and A, and also at C and E be equal one to the other; I say, their sides about the equal angles are proportionall; that is,

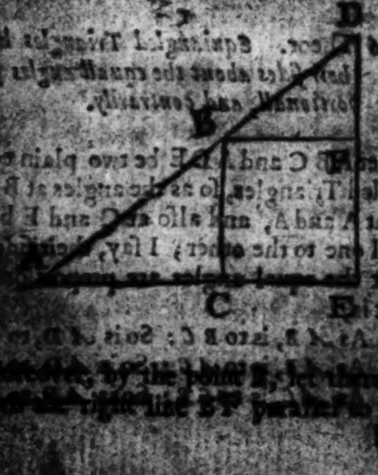
1 As *A B*, is to *B C*: So is *A D*, to *E D*.
2 As *A B*, is to *A C*: So is *A D*, to *A E*.
3 As *A C*, is to *C B*: So is *A E*, to *E D*.

De-

Demonstration.

Because the angles B A C and D A E are equall by the Proposition; therefore if A B be applied to A D, A C shall fall in A E; and by such application is this figure made. In which, because that A B and A D do meet together, and also that the angles at B and D are equall, by the Proposition; therefore the other sides B C and D E are parallel; and, by the last aforegoing, B C cutteth the sides A D and A E proportionally: and therefore,

As A B, to A D: So is A C, to A E.]



Moreover, by the point B, let there be drawn the right line B F parallel to the side

base A E, and it shall cut the other two sides proportionally in the points B and F, and therefore,

1. As A B to A D: so is E F to E D,

Or thus.

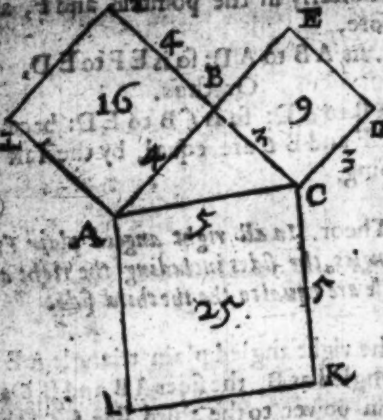
As A B to A D: so is C B to E D: because that F E and B C are equal, by the last aforegoing.

1. Theor. *In all right angled plain Triangles, the sides including the right angle are equal to the the third side.*

In the right angled plain triangle A B C, right angled at B, the sides A B and B C are equal in power to the third side A C; that is the squares of the sides A B and B C, to wit, the squares A L M B and B E D C added together, are equal to the square of the side A C, that is to the square A C K I.

Demonstration.

Let A B C be a triangle, right angled at B, and let the side B C be 3 foot, the side A B 4 foot, and the side A C 5 foot. Let every side be squared severally, so shall you finde the square of the side A C to contain as much as the squares of the sides A B and B C.

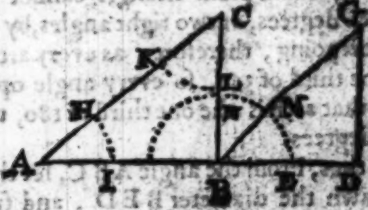


B C added together. For, the square of the side A B is 16, the square of B C is 9, which added together make 25, which is equall to the square of the side A C, which was to be demonstrated.

18. Theor. *The three angles of a right-lined Triangle are equal to two right angles.*

As in the following plain Triangle A B C the three angles A B C, A C B, and C A B are equal to two right angles. Let the side
A B

A B be extended to D, and let there be a semicircle drawn upon the point B, and let there be also dawn a line parallel unto A C, from B unto G.



Demonstration.

I say that the angle G B D is equal to the angle B A C, by the 9th hereof, and the angle C B G is equal to the angle A C B by the same reason, and the angles C B G and G B D, are together equal to the angle C B D, which is also equal to the angle A B C, by the 18th. of the first: and therefore; the three angles of a right lined Triangle are equal to two right angles, which was to be proved.

19. Theor. *If a plain Triangle be inscribed in a Circle, the angles opposite to the circumference are halfe as much as that part of the Circumference which is opposite to the angles.*

As if in the circle A B C the circumference B C be 120 degrees, then the angle B A C which is opposite to that circumference shall be 60 degrees. The reason is, because the whole circle A B C is 360 degrees, and the three angles of a plain triangle cannot exceed 180 degrees, or two right angles, by the last aforegoing, therefore, as every arch is the one third of 360, so every angle opposite to that arch is the one third of 180, that is 60 degrees.

Or thus, From the angle A B C, let there be drawn the diameter B E D, and from the center E to the circumference, let there be drawn the two Radii or semidiameters A E and A C, I say then that the divided angles A B D and D B C are the one halfe of the angles A E D and D E C: for the angles A B E and B A E are equall, because their Radii A E and E B are equall, and also the angle A E D is equal to the angles A B E and B A E added together, for if you draw the line E F parallel to A B, the angle F E D shall be equal to the angle A B E by the *9th.* hereof; and by the like reason the angle A E F is also equal to the angle B A E, and therefore the angle A E D is equal to the angles A B E, and B A E: or, which

which is all one, the angle A E D is double to the angle A B D.

In like manner, the angles E B C and E C B are equal, and the angle D E C is equal to them both: therefore the angle D E C is double to the angle D B C. Then because the parts of the angle A E C are double to the parts of the angle A B C, therefore also the whole angle A E C is double to the whole angle A B C; and thereupon the angle ABC is half the angle A E C; and consequently, half the arch A D C, is the measure of the angle A B C, as was to be proved. Hence it followeth,

 1. If

[illegible] If the side of a plain Triangle inscribed in a circle be the diameter, the angle opposite to that side is a right angle, that is, 90 degrees; for that it is opposite to a semicircle, which is 180 degrees.

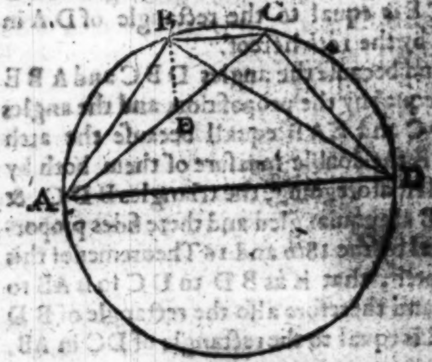
2. If divers right lined triangles be inscribed in the same segment of a circle upon one base; the angles in the circumference are equall. As the two triangles A BD and A C D being inscribed in the same segment of the circle A B C D, upon the same base A D are equiangled in the points B and C, falling in the circumference. For the same arch AD is opposite to both those angles; that is, to the angle A C D, and also to the angle A B D.

10 Theor. *If two plaine Triangles inscribed in the same segment of a circle, upon the same base, be so joyned together in the top, (or in the angles falling in the circumference) that thereof is made a four-sided figure, intersected with Diagonals, the right angled figure made of the Diagonals, is equall to the right angled figures made of the opposite sides added together.*

Let A B D and ACD be two triangles

 in-

ſcribed in the ſame ſegment of the circle A B C D upon the ſame baſe A D ſo joyned in the top by the right line B C, that thereupon is made the four ſided figure A B C D. I ſay, that the right angled figures made of the oppoſit ſides A B and D C, and alſo of the ſides B C and A D added together are equall to the right angled figure made of the Diagonals A C and B D.

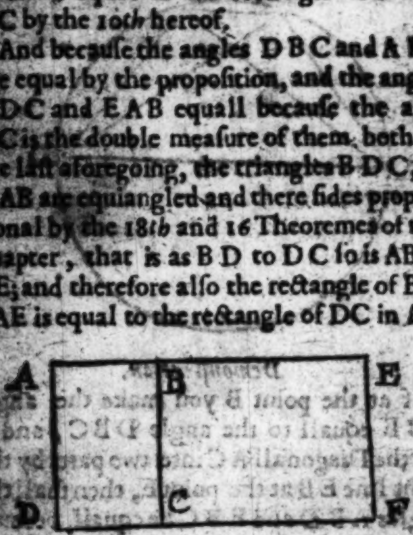


Demonſtration.

If at the point B you make the angle A B E equall to the angle D B C, and ſo cut the Diagonall A C into two parts by the right line E B at the point E, then ſhall the angles A B D and E B C be equall, becauſe the

the angles A B E and D B C are equal by the proposition, and the angle E B D common to both, and the angles A D B & E C B are equal, because the arch AB is the double measure of them both by the last aforegoing, and therefore the triangles A B D & E B C are equiangled and there sides proportional by the 18*th* and 16*th* Theoremes of this chapter, that is, as B D to D A, so is B C to CE, and therefore also the rectangles of B D in C E is equal to the rectangle of D A in B C by the 10*th* hereof.

And because the angles D B C and A B E are equal by the proposition, and the angles B D C and E A B equall because the arch B C is the double measure of them both by the last aforegoing, the triangles B D C, & E AB are equiangled and there sides proportional by the 18*th* and 16 Theoremes of this chapter, that is as B D to D C so is AB to AE; and therefore also the rectangle of B D in AE is equal to the rectangle of DC in AB.



And because the rectangled figure made of A D and D F is equall to the two rectangled figures of A D in D C and B C in C F, therefore also the rectangled figure of B D in A C is equal to the rectangled figures of B D in A E and B D in E C. From hence and the two former proportions the proposition is thus Demonstrated.

$$\left.\begin{array}{l}1.\ BD \text{ in } CE\\ 2.\ BD \text{ in } AE\end{array}\right\}\text{ is equal to }\left\{\begin{array}{l}DA \text{ in } BC\\ DC \text{ in } AB\end{array}\right.$$

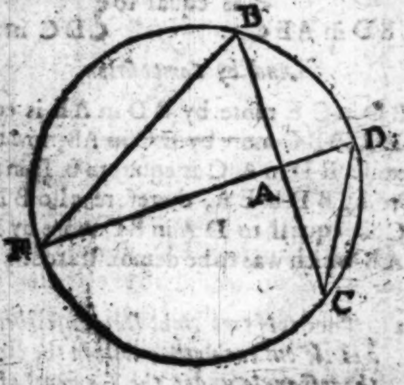
And by Composition.

B D in C E more by B D in A E is equal to D A in B C more by D C in AB, now then because B D in A C is equal to B D in in E more by B D in E A, therefore also B D in A C is equall to D A in B C more by D C in A B which was to be demonstrated.

21. Theor. *If two right lines inscribed in a circle cut each other within the circle, the rectangle under the segments of the one, is equall to the rectangle under the segment of the other.*

Let the two lines be F D and B C, intersecting each other in the point *A*; I say, the triangles *A* B F and *A* D C are like,

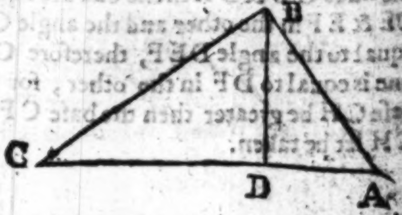
because of their equal angles *A* F B and *A* C D, which are equal, because the arch B D is the double measure of them both, and because of their equal angles B *A* F and D *A* C, which are equal by the ninth hereof, and where two are equal, the third is equal by the 18 aforegoing; therefore *A*D in *A*F is equal to *A*C in *A*B, which was to be proved.



Theor. 22. *In a plain right angled Triangle, a perpendicular let fall from the right angle upon the Hypothenuse, divides the triangle into two triangles, both like to the whole, and to one another.*

The

The triangle A B C is right angled at B, the hypotenuse or side subtending the right angle is A C, upon which from the point B is drawn the perpendicular B D which divideth the triangle A B C into two triangles, A D B and B D C, each of them like to the whole triangle A B C, and each like to one another also, that is equiangled one to another.



Demonstration.

In the triangle A B D, the angles A B D and A D B are equal to the angles A C B and A B C, because of their common angle at A, and their right angles at B and D, and in the triangle CDB, the angle. C B D and B D C are equal to the angles A B C and C A B, because of their common angle at C, and their right angles at B and D; these triangles are therefore each of them like to the whole triangle A B C, and by consequence like to one another.

24. *Theor. If two sides of one triangle be equal to two sides of another, & the angle comprehended by the equal sides equal, then the third side or base of the one, shall be equal to the base of the other, and the remaining angles of the one equal to the remaining angles of the other.*

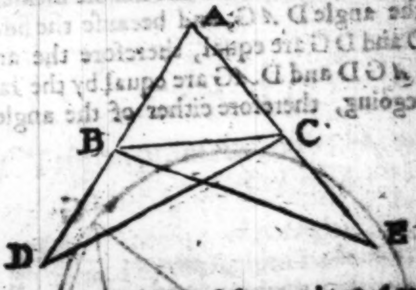
Of these two triangles C B H and D E F, the sides C B & B H inthe one are equal, to DE & E F in the other and the angle C B H equal to the angle DE F, therefore C H in one is equal to D F in the other, for if the base C H be greater then the base C F from C H let be taken.

C G equal to D F and let there be drawne the right line B G, now if B C and B G be equal to DE and E F, yet the angle C B G cannot be equal to the angle D E F by the an-

angle G B H which is contrary to the first position, and therefore C H must be equal to D F, and consequently the angle B C H equal to E D F, and C H B equal to D F E which was to be proved.

24. Theor. *An Isosceles or triangle of two equal sides, hath his angles at the base equal the one to the other, and contrariwise.*

Let the sides A B and A C in the triangle A B C be equal and produced at pleasure, so that *A* D may be equal to *A* E then draw the lines CD and BE; forasmuch as the two sides *A* D and *A* C in the triangle D *A* C are equal to the two sides A E and *A* B in

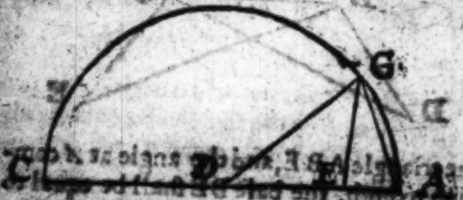


the triangle A B E, and the angle at *A* common to both, the base B E shall be equal to the base C D, and the angle at D to the angle at E, and the angle A B E to the angle A C D by the last aforegoing, likewise the

the angles D C B and E B C are equal, now if you take these equal angles from the equal angles A B E and A C D the angles remaining A B C and A C B must needs be equal, which was to be proved.

25. Theor. *If the Radius of a circle be divided, in extreame and mean proportion, the greater segment shall be the side of a Decangle, in the same circle.*

In the semicircle A G C let A G be the side of a Decangle D G or D A the Radius, then because the arch A G is the tenth part of a circle it is also the fift part of a semicircle, and the arch C G, which is four times as much as the arch A G is the double measure of the angle D A G, and because the sides A D and D G are equal, therefore the angles A G D and D A G are equal by the last aforegoing, therefore either of the angles



A G D or G A D is double to the angle A D G, now then if you divide the angle A G D into two equal parts by the right line E G the

angles E G D or E G A shall either of them be equal to the angle A D G and therefore E D & E G are equal by the last aforegoing, & the triangles A G D & A E G are equiangled because of their common angle D A G and there equal angles A G E and A D G, as before, and E G which is equal to D E is equal to A G, therefore as A D to A G (or E D) so is A G to A E, and the Radius A D is divided in extreame and meane proportion by the 14th hereof, and E D the greatest segment is the side of a decangle.

These foundations being laid we will proceed to the making of the tables, whereby any triangle may be measured.

Chap. III

of Trigonometria, or the measuring of all Triangles.

THe dimension of triangles, is performed by the Golden Rule of Arithmetick, which teacheth of four numbers proportionall one to another, any three of them being given, to finde out a fourth.

Therefore for the measuring of all triangles

[illegible] there must be certain proportions of all the parts of a triangle one to another, and these proportions must be explained in numbers.

2. And the proportions of all the parts of a triangle one to another cannot be certain unlesse the arches of circles (by which the angles of all triangles, and of Sphericall triangles, also the sides are measured) be first reduced into right lines, because the proportions of arches one to another, or of an arch to a right line, is not as yet found out.

3. The arches of every circle are after a sort reduced to right lines, by defining the quantity, which the right lines to them applied have, in respect of Radius, or the Semidiameter of the circle.

4. The arches of a circle thus reduced to right lines are either Chords, Sines, Tangents, or Secants.

5. A Chord or Subtense is a right line inscribed in a circle, dividing the whole circle into two segments, and in like manner subtending both the segments.

6. A chord or subtense is either the greatest or not the greatest.

7. The greatest Subtense is that which divideth the whole circle into two equal seg-

ſegments, as the right line G D, and it is alſo commonly called a diameter.

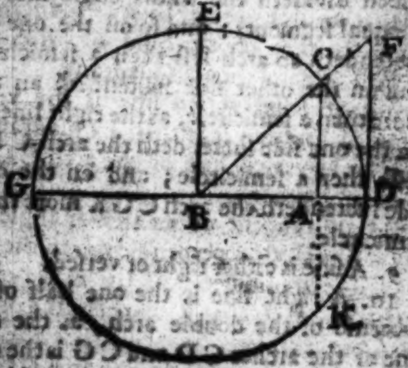
8. A ſubtenſe not the greateſt, is that which divideth the whole circle into two unequal ſegments: and ſo on the one ſide ſubtendeth an arch leſs then a ſemicircle; and on the other ſide ſubtendeth an arch more then a ſemicircle, as the right line C K on the one ſide ſubtendeth the arch C D K, leſſe then a ſemicircle; and on the other ſide ſubtendeth the arch C G K more then a ſemicircle.

9. A ſine is either right or verſed.

10. A right ſine is the one half of the ſubtenſe of the double arch, as the right ſine of the arches C D and C G is the right line *A C*, being half the chord or ſubtenſe of the double arches of C D and C G, that is, half of the right line C A K, which ſubtendeth the arches C D K and C G K: whence it is manifeſt, that the right ſine of an arch leſſe then a Quadrant, is alſo the right ſine of an arch greater then a Quadrant. For as the arch C D is leſſe then a Quadrant by the arch C E, ſo the arch C G doth as much exceed a Quadrant, the right line A C being the right ſine unto them both. And hence inſtead of the obtuſe angle G B C, which exceeds 90 degrees,

we

we take the acute angle C B A, the comple-ment thereof to 180: and so our Canon of sines doth never exceed a quadrant or 90 *d*.



11. Againe, a right sine is either *Sinus totus*, that is the Radius or whole sine, as in the triangle *A*B C, *A* C is the Radius, semidiameter, or whole sine, Or else the right sine is the *Sinus simpliciter*, that is, the first sine, as C*A* or B *A*, the one whereof is alwayes the complement of the other to 90 degrees; we usualy call them sine and co-sine.

12. The versed sine of an arch is that part of the diameter, which lieth between the right sine of that arch and the circum-ference. Thus *A D* is the versed sine of

the arch C D, and A G the versed Sine of the arch C E G, therefore of versed Sines some are greater, and some are lesse.

13. A greater versed Sine is the versed Sine of an arch greater then a Quadrant, as A G is the versed Sine of the arch C E G greater then a Quadrant.

14. A lesser versed sine is the versed Sine of an arch lesse then a Quadrant, as A D is the versed sine of the arch C D less then a Quadrant.

15. A tangent of an arch or angle is a right line drawn perpendicular to the Radius or semidiameter of the circle of the triangle, so as that it toucheth the outside of the circumference: And thus the right line F D is the tangent of the arch D C.

16. A secant is a right line proceeding from the center of the circle, and extended through the circumference to the end of the tangent; and thus B F is the Secant of the arch D C.

17. The definition of the quantity which right lines applyed to a circle have, is the making of the Tables of Sines, Tangents and secants, that is to say, of right Sines and not of versed; for the versed Sines are found by the right without any labour.

18. The lesser versed sine with the sine

of

of the complement is equal to the Radius; as the lesser versed sine A D with the right sine of the complement A B is equal to the Radius B D; therefore if you substract the right sine of the complement A B from the Radius B D, the remainder is the versed sine A D.

19. The greater versed sine is equal to the Radius added to the right sine of the excesse of an arch more then a Quadrant, as the greater versed sine A G is equal to the Radius B G with the sine of the excess A C: therefore if you adde the right sine of the excess AB to the Radius BG, you shal have the versed sine of the arch C E G, & so there is no need of the table of versed sines, the right sines may thus be made.

20. The Tables of Sines, Tangents, and Secants may be made to minutes, but may, by the like reason, be made to seconds, thirds, fourths, or more, if any please to take that paines: for the making whereof the Radius must first be taken of a certain number of parts, and of what parts soever the Radius be taken, the Sines, Tangents, and Secants are for the most part irrational to it, that is, they are inexplicable in any true whole numbers or fractions precisely, because there are but

few

few proportional parts to any Radius;
few proportional parts to any Radius, whose square root multiplied in it self will produce the number from whence it was taken, without some fraction still remaining to it; and therefore the Tables of Sines, Tangents, and Secants cannot be exactly made by any meanes; and yet such may and ought to be made, wherein no number is different from the truth by an integer of those parts, whereof the Radius is taken, as if the Radius be taken of ten Millions, no number of these Tables ought to be different from the truth by one of ten Millions.

That you may attain to this exactnesse, either you must use the fractions, or else take the Radius for the making of the Tables much greater then the true Radius, but to work with whole numbers and fractions is in the calculation very tedious; besides here no fractions almost are exquisitely true: therefore the Radius for the making of these Tables is to be taken so much the more, that there may be no errour, in so many of the figures towards the left hand as you would have placed in the Tables; and as for the numbers superfluous, they are to be cut off from the right hand towards the left after the ending of

the supputation, Thus, to finde the numbers answering to each degree and minute of the Quadrant to the Radius of 10000000 or ten millions, I adde eight ciphers more, and then my Radius doth consist of sixteen places.

This done, you must next finde out the right Sines of all the arches lesse then a Quadrant, in the same parts as the Radius is taken of, whatsoever bignesse it be, and from those right Sines the Tangents and secants must be found out.

21. The right Sines in making of the Tables are either primary or secondary. The primarie Sines are those, by which the rest are found, And thus the Radius or whole Sine is the first primary Sine, the which how great or little soever is equall to the side of a six-angled figure inscribed in a circle, that is, to the subtense of 60 degrees, the which is thus demonstrated.

Let BC be the side of a six angled figure inscribed in a circle, then because the arch B C is the sixt part of a circle, and that every cirle is suppoosed to be divided into 360 parts the side B C must needs be 60 parts, because six times 60 makes 360, and the angle B AC is 60 parts also, by the 23 of the first. And the angles ABC and

ACB

ACB are 120, by the 18 of the ſecond, and are alſo equal, becauſe the ſides AC and AB which are oppoſite unto them are equal, for they are two Radii, by the work, and therefore either of the angles are 60 parts, and conſequently the whole triangle is equiangled, and the whole triangle being equiangled, and the ſides AB and BC being Radii, the ſide BC muſt be Radius alſo. Therefore the Radius or whole Sine is equal to the ſide of a ſix angled figure inſcribed in a circle, as was to be proved.

Out of the Radius or ſubtenſe of 60 degrees the ſine of 30 degrees is eaſily found; the

the halfe of the subtense being the measure of an angle at the circumference opposite thereunto by the 19 of the second; if therefore your Radius consists of 16 places being 1000.0000.0000.0000. The sine of 30 degrees will be the one half thereof, to wit, 500.0000.0000.0000.

22. The other primary sines are the sines of 60, 45, 36, and of 18 degrees, being the halfe of the subtenses of 120, 90, 72, and of 36 degrees.

23. The subtense of 120 degrees is the side of an equilateral triangle inscribed in a circle, and may thus be found.

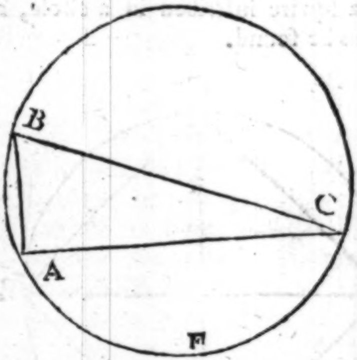
The Rule.

Substract the Square of the subtense of 60 degrees, from the Square of the diameter, the Square root of what remaineth is the side of an equilateral triangle inscribed in a circle, or the subtense of 120 degrees.

The reason of the Rule.

The subtense of an arch with the subtense of the complement thereof to 180 with the diameter, make in the meeting of the two subtenses a right angled triangle. As the subtense A B 60 degrees, with the subtense A C 120 degrees, and the diameter C B, make the right angled triangle A B C, right angled at A, by the 33 of the second,

And therfore the ſides including the right angle are equal in power to the third ſide, by the 17 of the ſecond. Therefore the ſquare of A B being taken from the ſquare of C B, there remaineth the ſquare of AC, whoſe ſquar root is the ſubtenſe of 60 degrees or the ſide of an equilateral triangle inſcribed in a circle,



Example.

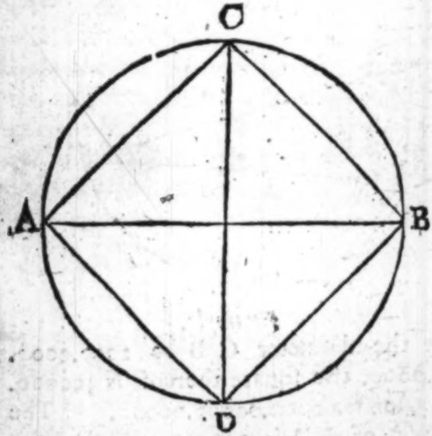
Let the diameter C B be 2000.0000.0000.0000. the ſquare thereof is 400000.00000.00000.00000.00000.00000. The ſubtenſe of A B is 100000.00000.00000. The ſquare thereof is 100000.00000.00000.00000.00000.00000; which being ſubſtract-

ed from the ſquare of C B, the remainder is 300000. 00000.00000.00000.00000.00000, whoſe ſquare root 173205.08075.68877. the ſubtenſe of 120 degrees.

CONSECTARY.

Hence it followeth, that the ſubtenſe of an arch leſſe then a Semicircle being given, the ſubtenſe of the complement of that arch to a Semicircle is alſo given.

24. The Subtenſe of 90 degrees is the ſide of a ſquare inſcribed in a circle, and may thus be found.



The Rule.

Multiply the diameter in it ſelf, and the ſquare

square root of half the product is the subtense of 90 degrees, or the side of a square inscribed in a circle.

The reason of this Rule.

The diagonal lines of a square inscribed in a circle are two diameters, and the right angled figure made of the diagonals is equal to the right angled figures made of the opposite sides, by the 20*th.* of the second, now because the diagonal lines A B and C D are equal, it is all one, whether I multiply A C by it self, or by the other diagonal C D, the product will be still the same, then because the sides A B, *A* C, and B C do make a right angled triangle, right angled at C, by the 19th. of the second, & that the sides *A* C and *C* B are equal by the work, the half of the square of *A* B must needs be the square of *A* C or C B, by the 17*th.* of the second, whose square root is the subtense of C B, the side of a square or 90 degrees.

Example.

Let the diameter *A* B be 200000.00000.00000, the square thereof is 400000.00000.00000.00000.00000.00000, the half whereof is 200000.00000.00000.00000.00000.00000, whose square root 141421.35623.73095. is the subtense of 90 degrees, or th

side of a square inscribed in a Circle.

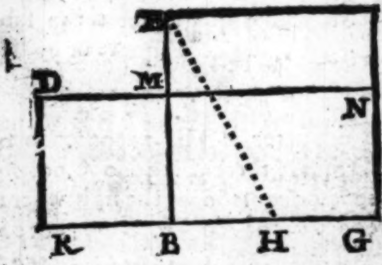
25. The subtense of 36 degrees is the side of a decangle, and may thus be found.

The Rule.

Divide the Radius by two, then multiply the Radius by it self, and the half thereof by it self, and from the square root of the summe of these two products substract the half of Radius, what remaineth is the side of a decangle, or the subtense of 36 degrees.

The reason of the rule.

In the following Diagram, let E B represent the Radius of a circle on which draw



... then is G B equal to E B,

which being bisected in the point H draw the line H E, then continue the segment H B to K, making H K equal to H E and upon the line K B make the square B D, then the Radius EB is divided into extreame and meane proportion by the 14*th* of the second, and the greater segment M B is the side of a decangle by the 25 of the second, and K B is equal thereunto; now then because the Radius E B and the half Radius H B with the right line H E, do make the right angled triangle E B H right angled at B, by the 21*th*. of the first, and therefore the squares of E B and B H are together equal to the square of H E or H K, by the 17*th*. of the second, now if from the square root of the square of H E, that is from the side H E or H K you deduct the side H B, the remainer is K B the side of a decangle.

For example.

Let the Radius EB be 100000.00000.00000 then is B H, or the half thereof 500000. 00000.00000. the square of E B is 100000 00000.00000.00000.00000.00000. and the square of B H 250000.00000.00000.00000. 00000.00000.00000. The summe of these two squares, *viz* 125000.00000.00000. 00000.00000. 00000. is the square of H E or

or H K, whose square root is 1118033988-749895, from which deduct the halfe Radius B H 500000000000000, and there remaineth 618033988749895, the right line K B, which is the side of a decangle, or the subtense of 36 degrees.

26 The subtense of 72 degrees is the side of a Pentagon inscribed in a circle, and may thus be found.

The Rule.

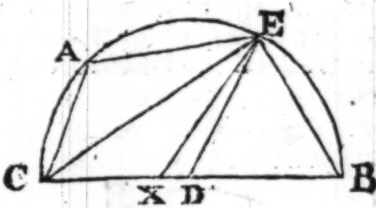
Substract the side of a decangle from the diameter, the remainer multiplied by the Radius, shall be the square of one side of a Pentagon, whose square root shall be the side it self, or subtense of 72 degrees.

The Reason of the Rule.

In the following Diagram let A C be the side of a decangle, equal to C X in the diameter, and let the rest of the semicircle be bisected in the point E, then shall either of the right lines A E or E B represent the side of an equilateral pentagon, for A C the side of a decangle subtends an arch of 36 degrees the tenth part of a circle, and therefore A E B the remaining arch of a semicircle is 144 degrees, the half whereof A E or E B is 72 degrees, the fift part of a circle, or side of an equilateral pentagon,

the

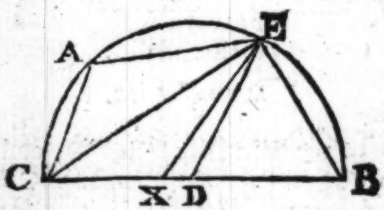
the ſquare whereof is equal to the oblong made of D B and B X.



Demonſtration.

Draw the right lines E X, E D, and E C, then will the ſides of the angles A C E and E C X be equal, becauſe C X is made equal to A C, and E C common to both; and the angles themſelves are equal, becauſe they are in equal ſegments of the ſame circle by the 19 of the ſecond; and their baſes A E and E X are equal by the 23 of the ſecond; and becauſe E X is equal to A E, it is alſo equal to E B, and ſo the triangle E X B is equicrural, and ſo is the triangle E D B, becauſe the ſides E D and D B are Radii, and the angles at their baſes X and B, E and B by the 24th. of the ſecond, and becauſe the angles at B is common to both, therefore the

the two triangles, E X B and E D B are equiangled, and their sides proportional, by the 18*th*. and 16*th*. Theoremes of the second Chapter, that is as D B to E B; so is E B to B X, and the rectangle of D B in B X is equal to the square E B, whose square root is the side E B, or subtense of 72 degrees.



Example.

Let A C, the side of a decangle or the subtense of 36 degrees, be as before: 618033988749895, which being substracted from the diameter B C 200000.00000.00000. the remainer is X B, 1381966011-251105, which being multiplied by the Radius D B, the product 1381966011251105 00000.00000.0000, shall be the square of E B whose square root 117557050458494 6 is

Is the right line E B, the side of a Pentagon or subtense of 72 degrees.

CONSECTARY.

Hence it followes, that the subtense of an arch lesse then a semicircle being given, the subtense of half the complement to a semicircle is given also.

Thus much of the primarie Sines, the secondary Sines or all the Sines remaining may be found by these and the Propositions following

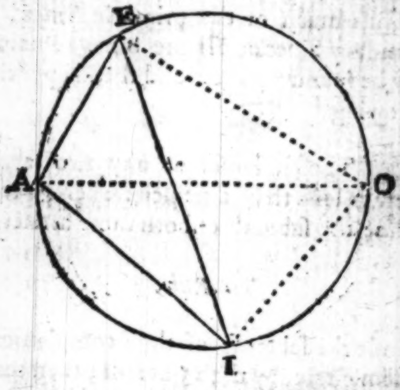
27. The subtenses of any two arches together lesse then a semicircle being given, to finde the subtense of both those arches.

The Rule.

Finde the subtense of their complements to a semicircle, by the 23 hereof; then multiply each subtense given by the subtense of the complement of the other subtense given, the sum of both the products being divided by the diameter, shall be the subtense of both the arches given.

The reason of the Rule.

Let the ſubtenſes of the given arches be the right lines A E and A I, and let the ſubtenſe of both thoſe arches be the right line E I, let the diameter A O be drawn to the



very point in which the ſubtenſes of the given arches do concurre, to wit, in the point A. Then draw the right lines E O and I O, which with the diameter and the ſubtenſes given, do make the two right angled Triangles A E O and A I O, right angled at E and I (by the 19*th* of the ſecond.)

And

And therefore the ſides E O and I O are given by the 23 hereof, and conſequently the right angled figures made of A E and I O, A I and E O, to which the right angled figure made of the diagonals E I and A O is equal by the 20th. of the ſecond, and therefore the ſumme of the right angled figures made of A E and I O, and alſo of A I and E O, being divided by the diameter A O, the quotient is E I, the ſubtenſe of both the arches given.

Example.

Let A I, the ſide of a ſquare or ſubtenſe of 90 degrees be 141421.35623.73059. And E O, the ſide of a triangle, or ſubtenſe of 120 degrees, 173205.08075.68877, the product of theſe two will be 24494897 4278-3 77659465841164315. Let A E, the ſide of a ſixangled figure, or the ſubtenſe of 60 degrees be 100000.00000.00000. And I O, the ſide of a ſquare, or ſubtenſe of 90 degrees 141421.35623.73059 the product of theſe two will be 141421.35623.73059.00000.00000.00000. the ſumme of theſe two products 38637033051562716594658-44164315. And this ſumme divided by the diameter A O, 200000.00000.00000. leaveth

veth in the quotient for the ſide E I, or ſubtenſe of 150 degrees, 193185165257813б. the half whereof 96592582628906 8, is the Sine of 75 degrees.

28 The ſubtenſes of any two arches leſſe then a Semicircle being given, to finde the ſubtenſe of the difference of thoſe arches.

The Rule.

Finde the ſubtenſes of their complements to a ſemicircle, by the 23 hereof, as before; then multiply each ſubtenſe given, by the ſubtenſe of the complement of the other ſubtenſe given; the leſſer product being ſubſtracted from the greater, and their difference divided by the diameter, ſhall be the ſubtenſe of the difference of the arches given.

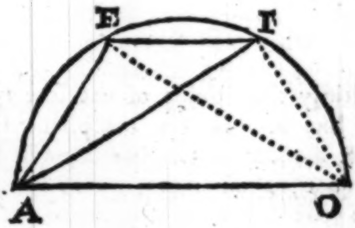
The Reaſon of the Rule.

Let the ſubtenſes of the given arches be A E and EI, and let the ſubtenſe ſought be the right line E I; then becauſe the right angled figure made of the diagonals A I and E O is equal to the right angled figure made

made of their oppoſite ſides, by the 20 of the ſecond; therefore if I ſubſtract the right angled figure made of *A* E and I O, from the right angled figure made of *A*I and E O the remainer will be the right angled figure of *A* O and E I, which being divided by the diameter *A* O, leaveth in the quotient E *A*.

Example,

Let the right angled figure *A* I and



E O be the ſame with the former, *viz.* 2449489742783177659465844164315. And the right angled figure of *A* E and I O 141421356237305 9. 00000. 00000. 00000. Their difference ſhall be 1035276180410082659465844164315, which divided by the diameter *A* O, leaveth in the quotient

517

5176380902050041, for the subtense of the difference of the arches of 60 and 90, that is, for the subtense of 30 degrees. The half whereof, *viz.* 2588190451025020, is the sine of 15 degrees

29. The sine of an arch lesse then a Quadrant being given, together with the sine of half his complement, to finde the sine of an arch equal to the complement of the arch given, and the half complement added together.

The Rule.

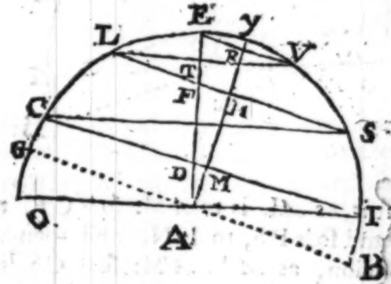
Multiply the double of the sine given, by the sine of half his complement, the product divided by the Radius, will leave in the quotient, a number, which being added to the sine of the half complement shall be the sine of the arch sought.

The reason of the Rule.

Let E A I be a quadrant, and in that let the arches I S, S V, V E be equal, then let the last arch V E be bisected in Y, and let the Quadrant be made into a Semicircle, and the arches O C, C L, L E, equall

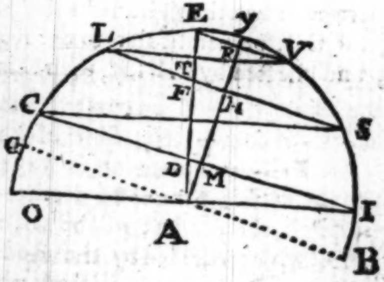
to

to the former: then shall the right lines L V and C S be parallels to the diameter O A I, and bisected by the Radius A E, and because Y V is half of the arch E V, it is also the half of the arch V S or S I, and equal to the arches I B, C G, or G O, Then let there be drawn the right lines E V, L S, C I, and G B, perpendicular to the Radius A Y, and bisected by it. I say, then that the



right angled triangles I A M, C P M, and S P N are equiangled, for the arches C O, G L, and S I are equal, by the work, and the double measures of the angles A I M, P C M, and P S N, and the angles A M I, C M P, and P N S are equall, that is, right angles, because the right line A Y doth fall perpendicularly upon the parallel right lines

lines L S and C I, and now where two angles are equal, there the third is equall, by the 18*th*. of the second; and consequently the sides of the triangles I *A* M, C P M, and S P N are proportional.



That is, as *A* I, is to *A* M: so is C P, to P M; and so is P S, to P N, and then by composition, as *A* I, *A* M: so is C S, to M N. Now then let E S be the arch given, and S I the complement thereof to a Quadrant, then is C G or I B, being equal to E Y, the half of the said complement S I, and *A* M is the Sine thereof, and the Sine of E S is the right line H S, and the double C S, M N is the difference between *A* M, the Sine of C G or I B, and A N the Sine of S B, and *A* I is the Radius, and it is al-

ready

ready proved, that *A* I is in proportion to *A* N, as C S, is to M N; therefore if you muliply *A* M by S C, and divide the product by *A* I, the quotient will be N M, which being added to *A* M, doth make A N, the Sine or the arch ſought.

Example.

Let E S, the arch given, be 84 degrees, and the Sine thereof 9945219, which doubled is 19890438, the Sine of 3 degrees, the halfe complement is 523360, by which the double Sine of 84 degrees being multiplied, the product will be 10409859.631680, which divided by the Radius, the quotient will be 10409859, from which alſo cutting off the laſt figure, becauſe the Sine of 3 degrees was at firſt taken too little, and adding the remainer to the Sine of 3 degrees, the aggregate 1564345 is the Sine of 6 degrees, the complement of 84, and of 3 degrees, the halfe complement added together, that is, it is the ſine of 9 degrees.

30. The ſubtenſe of an arch being given, to find the ſubtenſe of the triple arch.

The

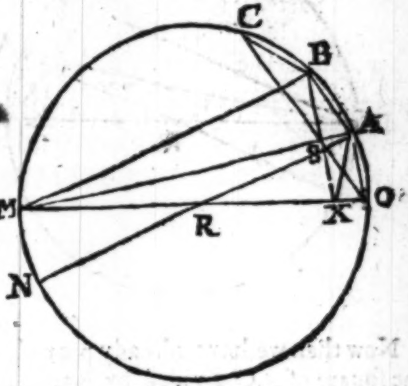
The Rule.

Mutiply the ſubtenſe given by thrice Radius ſquare, and from the product ſubſtract the cube of the ſubtenſe given, what remaineth ſhall be the ſub tenſe of the triple arch.

The reaſon of the Rule.

If in a circumference you diſtinguiſh three equall parts from O the end of the Diameter, with the letters AB C and draw the ſubtenſes as in the ſcheame, making M X equal to M B D, drawing alſo A X and A B and the diameter N R A, then ſhall the triangles BMX and A R O be equicrurall becauſe R A and R O are two Radii, and M B and M X are equal by the worke, and the angles B M X and A R O are equal by the 19*th* of the ſecond; and therefore the triangles B M X and A R O are equiangled by the 23 of the ſecond, and becauſe the ſides MB and M X are equal and A M common to both the triangles A M B and A M X, therefore AX is equal to A B by the 24*th* of the ſecond, and A B is equal to A O by the work, and therefore A X is alſo equal to A B, and the angles A X O and A O X are equal by the 24*th* of the

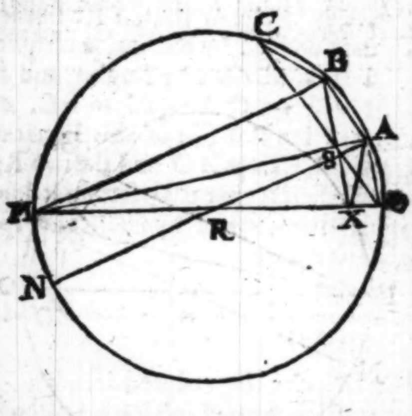
the second, and the riangles A O X and A R O are like, because the angle A O R is common to both & therfore as AR to AO, so is A O to O X; that is A O square divided by Radius, is equal to O X and O S is equal to A O and OX to A S, because the triangles A O X and A O S are equiangled, the angles S A O and A O X are equal because they are the same with two of the angles in the equiangled triangle A R O; and the angles A O S and X A O are equal, because



they are measured by equal arches, for A C the double of A O, is the double measure of the angle A O S, by the nineteenth of the second, and A O is the measure of

A R O

A R O equal to X A O, becauſe the triangles A R O and A O X are like. And then becauſe A S is equal to O X, S N muſt needs be equal to M X or M B, and the right angled figure made of O S and S C, is equal to the right angled figure made of A S and S N, by the 21*th.* of the ſecond, that is, as O S, to N S, ſo is S *A* to S C.



Now then we have already proved, that the ſquare of *A* O divided by Radius, is equal to O X, and alſo that O X is equal to S *A*, and therefore S N is leſs then twice Radius by the right line *A* S; or thus, N S is twice Radius leſs by *A* O ſquare divided by

by Radius; and N S multiplied by S *A* is the ſame with twice Radius leſſe by *A* O ſquare divided by Radius, multiplied into *A*O ſquare divided by Radius, and NS multiplied by S *A* is equal to S C multiplied by O S; and therefore twice Radius leſs *A* O ſquare divided by Rad. multiplied by *A* O ſquare divided by Radius, is equal to S C, multiplied by S O: or thus, 2 Radius leſs *A O* ſquare divided by Radius, multiplied into *A* O ſquare divided by Radius, and divided by *A* O or S O is equal to S C. All the parts of the firſt ſide of this Equation are fractions, except *A* O and the two Radii, as will plainly appear, by ſetting it down according to the form of Symbolical or ſpecious Arithmetick; thus.

$$2\text{Rad.} - \frac{AO\,aa}{\text{Rad,}} \text{ into } \frac{AO\,aa}{\text{Rad.}} \text{ divided by } AO$$

$=$ S C. Which being reduced into an improper fraction, by multiplying 2 Radius by Radius, the Equation will run thus:

$$\frac{2\,\text{Rad.}aa - AO\,aa}{\text{Rad.}} \text{ into } \frac{AO\,aa}{\text{Rad.}} \text{ divided by } AO = \text{S C.}$$

And then theſe two fractions having one common denominator, they may be reduced

ced into one after the manner of vulgar fractions, that is, by multiplying the numerators, the product will be a new numerator, and by multiplying the denominators the product will be a new denominator; thus multiplying the numerators, 2 Rad. aa -- $AO\,aa$ by the numerator $AO\,aa$, the product is 2 Rad. square into AO square, less AO square square, as doth appear by the operation;

$$\begin{array}{l} 2\,\text{Rad.}aa - AO\,aa \\ \quad AO\,aa \\ \hline 2\,\text{Rad.}aa \times AO\,aa - AO\,aaaa \end{array}$$

And then the denominators being multiplied by the other, that is, Radius being multiplied by Radius, the product will be Radius aa for a new denominator, and then the Equation will run thus;

$$\frac{2\,\text{Rad.}\,aa \times AO\,aa - AO\,aaaa}{\text{Radius}\,aa}$$ divided

by $AO = SC$: but before this fraction can be divided by AO, AO being a whole number, must be reduced into an improper fraction, by subscribing an Unite; and then the Equation will be;

$\frac{2\,Rad.aa \times AO\,aa -- AO\,aaaa}{Rad.aa}$ divided by $\frac{AO}{1} = SC$. Now as in vulgar fractions, if you multiply the numerator of the dividend by the denominatos of the diviſor, the product ſhall be a new numerator; again, if you multiply the denominator of the dividend by the numerator of the diviſor, their product ſhall be a new denominator, and this new fraction is the Quotient ſought in this example, the numerator will be ſtill the ſame, and the denominator will be Radius ſquare multiplied in A O, and the fraction will be

$\frac{2\,Rad.aa \times AO\,aa -- AO\,aaaa}{Rad.aa \times AO.}$ And in its leaſt termes it is

$\frac{2\,Rad.aa \times AO -- AO\,aaa}{Rad.aa} = SC$. In words thus: Twice Radius ſquare multiplied in A O, leſſe by the cube of A O divided by Radius ſquare is equal to S C. And by adding A O to both ſides of the Equation, it will be, twice Radius ſquare in A O, leſſe A O cube divided by Radius ſquare, more A O, is equal to S C more A O, that is, to

OC.

O C. Here again *A* O, the last part of the first side of this Equation is a whole number, and must be reduced into an improper fraction, by being multiplied by Radius square, the denominator of the fraction; and then it will be Radius square in *A* O divided by Radius square, which being added to twice Radius square in *A* O, divided by Radius, the summe will be 3 Radius square in *A* O divided by Radius square, and the whole Equation

$$\frac{3\,\text{Rad.}aa \text{ in } AO - AO\,aaa}{\text{Rad.}aa} = \text{O C},$$ the subtense of the triple arch.

For Example.

Let *A* O or *A* B, 17431.14854.95316, the subtense of 10 degrees be the subtense given, and let the subtense of 30 degrees be required; the Radius of this subtense given consists of 16 places, that is, of a unite and 15 ciphers, and therefore thrice Rad. square is 3, and 30 ciphers thereunto annexed, by which if you multiply the subtense given, the product will be 52293.44564.85948. 00000. 00000. 00000. 00000. 00000. 00000. the square of this subtense given is 30384-49397.55837.60253.85793.9856, and the cube 529.63662.80907.48519.77452.00270. 23094.54977.14496, which being substracted from

from the former product, there will remain 51763.80902,05040.51480,52567.59725.76005,45022.85504. this remainder divided by the square of Radius, will leave in the quotient, 51763.80902,05040. for the subtense of 30 degrees.

31. The subtense of an arch being given, to finde the subtense of the third part of the arch given.

The Rule.

Multiply the subtense given by Radius square, and divide the product by thrice Radius square, subtracting in every operation the cube of the figure placed in the quotient from the triple thereof, so shall the quotient in this division be the subtense of the third part of the arch given.

The reason of the Rule.

The reason of the rule is the same with the triple arch, but the manner of working is more troublesome, the which I shall endevour to explain by example.

Let there be given the subtense of 30 degrees, 517638090205040, and let the subtense of 10 degrees be required: First, I multiply the subtense given by the square

of Radius, that is, I adde 30 ciphers thereunto, and for the better proceeding in the work, I distinguish the subtense given thus inlarged by multiplication into little cubes, setting a point between every third figure or cipher, beginning with the last first, and then the subtense given will stand thus: 517.638.090.205.040.000.000.000.000.000 000.000.000.000.000. And so many points as in this manner are interposed, of so many places the quotient wil consist, the which in this example is 15, and because here are too many figures to be placed in so narrow a page, we will take so many of them onely as will be necessary for our present purpose; as namely, the 15 first figures, which being ordered, according to the rules of decimal Arithmetick, may be divided into little cubes, beginning with the first figure, but then you must consider whether the number given to be thus divided be a whole number or a fraction, if it be a whole number, you must set your point after or over the head of the first figure, if it be a fraction, place as many ciphers before the fraction given, as will make it consist of equal places with the denominator of the Fraction given; thus the subtense given being a fraction, part of the supposed Radius

of

of a circle, the which, as hath been said doth consist of 16, and the subtense given but of fifteen, I set a cipher before it, and distinguish that cipher from the subtense given by a point or line, and every third figure after, so will the subtense given be distinguished into little cubes, as before. This done, I place my divisor thrice Radius square, that is, 3 with ciphers (or at least supposing ciphers to be thereunto annexed) as in common division under the first figure of the subtense given, that is, as we have now ordered it under the cipher, and ask how often 3 in nought, which being not once, I put a cipher in the margine, and move my divisor a place forwarder, setting it under 5, and ask how often 3 in 5, which being but once, I place one in the quotient, and the triple thereof being 3, I place under 3 my divisor, and the cube of the figure placed in the quotient, which in this case is the same with the quotient it self, I set under the last figure of the first cube, and supposing ciphers to be annexed to the triple root, I substract this cube from it, and there doth remain 299, which is my divisor corrected; with this therefore I see whether I have rightly wrought or not, by asking, how often 299 is contained in the

first cube of the subtense given, 517, which being but once, as before, the former work must stand, & this divisor corrected must be subtracted from the first cube in the subtense given, and there will rest 218, and so have I wrote once. To this remainer of the first cube 218, I draw down 638, the figures of the next cube & moving my divisor a place forwarder, I ask, how often 3 in 21, which being 7 times, I put 7 in the quotient, and under the first figure of this second cube, that is, under 6 I set the triple square of the first figure in the quotient, that is, 3, for the quotient being but one, the square is no more, and the triple thereof is 3; under the second figure of this second cube I set the triple quotient, the which in this example is likewise 3, and both these added together, do make 33, which being substracted from my divisor 3000, there will remain 2967, for the divisor corrected, and by this also I finde the quotient to be 7, and yet I know not whether my work be right or not, I must therefore proceed, and set the triple of the figure last placed in the Quotient under the first figure of the remainer of the first cube, that is, I must set 21, the triple of 7 under 2, the first figure of 218, and now having two figures in the quo-

quotient, for distinction sake I call the first *a*, and the second *e*, that so the method of the work may the better be seen in the margine, and I set 3 *a a e*, that is, 3, the square of the first figure noted with the letter *a*, *viz*. 1. multiplied by the second figure, noted with the letter (*e*) to wit, 7, under the first figure of the next cube, now the square of (*a*) that is, of one is one, and the triple of this square is 3, and 3 times 7 is 21, which is (3 *a a e*) or, thrice (*a*) square in *e*, the last figure whereof, to wit, one, I place under 6, the first figure of the next cube 638: next I set (3 *a e e*) that is, three times one multiplied by the square of 7, that is, 3 multiplied by 49, which is 143 under the 2 figure of the cube 638: and lastly, I set (*eee*, that is) the cube of *e*, that is, the cube of 7, *viz*. 343, under the last figure of the cube 638, and these 3 sums added together do make 3873, which being substracted from the triple root, that is, from 21, supposing ciphers to be thereunto annexed, as before, there will remain 206127, and because this may be subtracted from the 2d. cube, & the remainer of the first, I finde that 7 is the true figure to be placed in the quotient, and such a subtraction being made, the remainer will be 12511, and so have I wrote twice.

 The

The work following must be done in all things, as this second, save onely in this particular, that both the figures in the quotient are reckoned but as one, which for distinction sake I called *a*, and the figure to be found by division I called *e*, and therefore in this third work 3 *a a*, or thrice *a* square is the square of 17, that is 289, 3 *a* or thrice *a* is 3 times 17, that is, 51, and so of the rest, in the fourth work the three first figures must be called *a*, in the fifth work the four first figures found, and so forward, till you have finished your division, and therefore this second manner of working being well observed, there can be no difficulty in that which followes.

The Quotient   1 7 4 3 1

a   e

0	517	638	090	305	040	917	
8	3··	···	···	···	···	cc	
	3··					−cca	
	··1					−aaa	
	299						
Rest	218	638					
	·3·	···				+ec	
		3··				−3aa	
		03·				−3a	
		33·					
	·29	67·				Divisor	
	21·	···				+eee	
	2	1··				−3aae	
	1	47·				−3aee	
		343				−eee	
	3	913					
	206	087					
Rest	012	551	09[illegible]				

Resi	12	551	090	205	040	4 Quotient
		...	...			+ cc
		·86	7..	...	...	−3aa
			51·			−3a
		·87	21·			1..
	2	912	79·			Divisor
	12	...	...			+ ccc
		346	8..			−3aac
		8	16·			−3acc
			64			−ccc
		355	024			
	11	644	976			Substract

Resi	906	114	205	..	3 Quotient
	3..	...	...		+ cc
	9	082	8..		−3aa
		5	32·		−3a
	9	088	02		
	290	911	98		Divisor

	290	911	98		*Divisor*
	9..	...	...		+ccc
	27	248	4..		--3aae
		46	98.		--3aee
			.27		--eee
	27	295	407		
	872	704	593		*Substract*
Rest	33	409	712	040	1 *quotient*
	3..	...	...	...	+ccc
		911	414	7..	3aa
			52	29.	3a
		911	466	99.	
	29	088	533	01	*Divisor*
	3..	...	...	...	+ccc
		911	414	7..	-3aae
			52	29.	-3aee
				1	-eee
		911	466	991	
	29	088	533	009	*Substract*
Rest	4	321	179	031	

38. The subtense of an arch being given to finde the subtense of the arch quintuple, or of an arch five times as much.

The Rule.

From the product of the subtense given, multiplied by 5 times Radius square square, subtract the cube of the subtense given multiplied by 5 times Radius square, the squared cube of the subtense given being first added thereunto, the remainer divided by Radius square square, shall leave in the quotient the subtense of the arch quintuple, or the arch 5 times as much.

The reason of the Rule.

In the annexed Diagram, twice E T more C B is equall to O E, because O E is the subtense of five equall arches, by the work, and by letting fall the perpendicular C T, the right line O T doth answer to three equall arches, A O, A B, and B C; and therefore E T doth answer to the other two: now if you deduct the right line C B from the right line O T, the remainer must be equall to E T, and so it followes, that 2 E T + C B = O E. And the triangles O B M, and O C T, are equiangled, because of their equall angles C T O and M B O, which are both

both right, and the angles B M O & C O T are equal, because they are measured by equal arches; and therefore, as M O is to M B: so is O C, to T O: that is, as hath been shewed in the triplication of an angle. As twice Radius, is to twice Radius, lesse by the square A O divided by Radius: so is thrice Radius square in A O, less by the cube of A O divided by Radius square, to a fourth number represented by the right line O T, what that number is by the rule of proportion may be thus found:

Multiply the numerator of the fractions in the second place by the numerator of the

the fraction in the third, and their product will be a new numerator, the numerator of the fraction in the second term is

2 Rad. — AO aa

And in the third, 3 Rad. aa × AO — AO aaa

That one of these termes may be the better multiplied by the other, the first of the second term, 2 Rad. must be reduced into an improper fraction, by the multiplication thereof by Radius, the denominator of that fraction, and then the 2d. term will be 2 Rad. aa — AO aa, and because this second term is the lesse, we will multiply the third thereby, the work stands thus:

3 Rad. aa × AO — AO aaa
2 Rad. aa — AO aa

6 R. aaaa × AO — 2 R. aa × AO aaa
— 3 R. aa × AO aaa + AO aaaaa

6 R. aaaa × AO — 3 R. aa × AO aaa + AO aaaaa.

Thrice Radius square in AO multiplied by twice Radius square, doth make 6 Rad. square squares, and AO cube multiplied by 2 Radius square is 2 Rad. square in AO cube, and because it hath the signe lesse, therefore the first product is 6 Rad. aaaa × AO — 2 Rad. aa × AO aaa. Again, 3 R. aa in AO, multiplied by AO aa, doth make [illegible] in AO aaa; & AO aaa multiplied

by

by AO*aaa*, doth make AO*aaaaa*, & because it hath the sign less, therefore the 2. product is 3 R. square × AO*aaa* + AO*aaaaa*, and so both the products will be 6 Rad. square of squares multiplied by A O lesse by 5 Rad. square in A O cube more by A O square cube. And if you multiply Rad. square, the denominator of the third term by Rad. the denominator of the second, the product will be Rad. cube, and the whole product will stand thus,

$$\frac{6R.aaaa \times AO - 5R.aa \times AOaaa + AOaaaaa.}{Radius\ aaa}$$

To divide this product by twice Radius, twice Radius being a whole number must be first reduced into an improper fraction, by subscribing an unite thus, $\frac{2\ Radius.}{1}$ then if you multiply the numerator of the product by one, the denominator of this fraction, the product will be still the same, and if you multiply the denominator of the product Rad. *aaa* by 2 Radius, the numerator of this improper fraction, the product will be 2 Rad. square square for a new denominator, and the Quotient will be

$$\frac{6R.aaaa \times AO - 5R.aa \times AOaaa + AOaaaaa}{2\ Radius\ aaaa}$$

the quantity of the right line OT, the double whereof is

$$\frac{12R,aaaa \text{ in } AO - 10R,aa \text{ in } AOaaa - 2AOaaaaa.}{2 \text{ Rad},aaaa}$$

which is the quantity of the right line OE more by C B, and therefore C B or A O being deducted, the remainer will be the right line OE, which is the quintuplation of an angle, and to this end A O must be reduced into an improper fraction of the same denomination, that is, by multiplying thereof by 2 *Rad,aaaa*, and then the fraction will be $\frac{2 \text{ Rad},aaaa \text{ in } AO}{2 \text{ Rad},aaaa}$ and this being deducted from

$$\frac{12R,aaaa \text{ in } AO - 10R,aa \text{ in } AOaaa - 2AOaaaaa.}{2 \text{ Rad},aaaa}$$

the remainer will be

$$\frac{10R,aaaa \text{ in } AO - 10R,aa \text{ in } AOaaa - 2AOaaaaa.}{2 \text{ Rad},aaaa.}$$

And this reduced into its least terms, will be

$$\frac{5R,aaaa \text{ in } AO - 5R,aa \text{ in } AOaaa - 1AOaaaaa.}{\text{Rad},aaaa}$$

$= OE$, which was to be proved.

For example.

Let *AG* or *A* B 349048, the subtense of 2 degrees be given, and let the subtense of 10 degrees be demanded, 5 times *Radius* square square is 5000000.0000000.0000000 00.0000000. by which if you multiply the subtense given, the product will be 1745240 0000000.0000000.0000000.0000000. The Cube of the subtense given multiplied by 5 times Radius square is 21263045378199-2960.0000000.0000000, the squared cube of the subtense given is 51846392428249-21385360723968, the which being added to the product of 5 *Radius* in *AG*, that is, to 21263045378199 2960.0000000.0000000 the summe will be 21268230017444 1209-21385360723968. And this being subtracted from the product of the subtense given multiplied by 5 times Radius square square, the remainer will be 1743 114176998255 7-8790786146391 76032, and this remainer divided by *Radius* square square, that is, cutting off 28 figures, their quotient will be 1743114, the subtense of 10 degrees.

33. The subtense of an arch being given to finde the subtense of the fift part of the arch given.

The Rule.

Divide the subtense given by five roots, lesse

lesse 5 cubes, more one Quadrato cube, the quotient shall be the fift part of the arch given.

The reason of the rule depends upon the foregoing Probleme, in which we have proved, that the subtense of five equall arches is equall to 5 roots, lesse 5 cubes, more by one quadrato cube, of which 5 roots one of them is the subtense of the fift part of the arch given: And consequently, if I shall divide the subtense of five equall arches by 5 roots, lesse 5 cubes, more one quadrato cube, the quotient shall be the subtense of the fift part of the arch.

The manner of the work is thus: First, consider whether the subtense given to be divided doth consist of equal, or of fewer places then the *Radius* thereof, if it consist of equal places, set a point over the head of the first figure of the subtense given; if of fewer places, make it equal, by prefixing as many ciphers before the subtense given as it wanteth of the number of places of the *Radius* thereof.

For example.

Let the subtense of 10 degrees be given, viz. 0.17431.14858.93316.34711. This is lesse then the *Rad.* by one place, and therefore

fore I have ſet one cipher before, and have diſtinguiſhed it from the ſubtenſe given by a point ſet between, the which is all one, as if it had been put over the head thereof: next you muſt diſtinguiſh the ſubtenſe given into little cubes, & into quadratocubes, which may be conveniently done thus; having found the place of the firſt point, which is alwayes the place of the *Radius*, the ſubtenſe given muſt be diſtinguiſhed into little cubes, by putting a point under every third figure, as in the triſection of an angle: thus in this example the firſt cubick point will fall under the figure 4, and the ſubtenſe given muſt be diſtinguiſhed into quadrato cubes, by ſetting a point over the head, or elſe between every fift figure from the place of the *Radius*: thus in this example the firſt quadrato cubick point muſt be ſet over the head, or after the figure of 1, the ſecond after 4, as here you ſee.

After this preparation made, you muſt place your two diviſors, 5 roots and 5 cubes in this manner, the firſt as in ordinary diviſion under the firſt figure of the ſubtenſe given, the other 5 under the firſt cubick point, and they will ſtand as in the work you ſee; then ask how often 5 in one, which being not once, I put a cipher in the

quo-

quotient, and remove my first divisor a place forwarder, as in ordinary division, but the other 5 I remove to the next cubick point, then, as before, I ask how often 5 in 17, which being 3 times, I set 3 in the quotient, and of this quotient I seek the quadrato cube, and finde it to be 243, the last figure whereof, namely, 3, I set under the last figure of the second quadrato cubick point (because there are but 3 figures between my divisor 5 and the first cubick point, whereas there must be alwayes four at the least) then I multiply the figure 3 placed in the quotient by my divisor 5, and the product thereof is 15, the first figure whereof I place under my said divisor 5, to which having annexed ciphers, or at least supposing them to be annexed, (as to the triple root in the trisection) I draw the quadrato cube of the figure in the quotient, and these 5 roots or 5 quotients into one summe, the which is 1500000243, under this summe I draw a line, so have we five roots more one quadrato cube, from which I must substract 5 cubes, I therefore seek the cube of 3, the figure placed in the quotient, and finde it to be 27, which multiplied by 5, the product will be 135, the last figure of these five cubes, *viz*. 5, I set under my second

cond 5 or cubick divisor, and substracting these 5 cubes from the 5 roots more one quadrato cube, the remainer will be 148650243, which remainer being also substracted from the figures of the subtense given standing over the head thereof, the remainer of the subtense given will be 244464611, and so have I wrought once.

To this remainer of the two first quadrato cubes, I draw down 95316, the figures of the next quadrato cube, and setting my first divisor a place forwarder, I ask how often 5 in 24, which being four times, I set 4 in the quotient, not knowing yet whether this be the true quotient or not, but with this I proceed to correct my divisor, and first I seek the quadrato quadrat of 3, the first quotient, and finde it to be 81, this multiplied by 5, will make 405, this product I set under my divisor, and 5 the last figure thereof I set under 9, the first figure of the 3 quadrato quadrate; next I seek the cube of 3, & finde it to be 27, which being multiplied by 10, the product will be 270, and this I set a place forwarder under the former product: thirdly, I seek the square of 3, which is 9, and this multiplied by 10 is 90, which I set a place forwarder under the second product 270. Lastly, I multiply 3,

the

the figure in the quotient by 5 my divisor, this product which is 15, I set a place forwarder under 90, the third product, and now these 4 products together with my divisor and ciphers thereunto annexed, being gathered into one summe, will be 500000-432915, under which I draw a line. And thrice the square of 3, multiplied by 5, which is 135, I set under this summe, the last figure thereof 5, under the first figure of the third cubick point, that is, under 4, and the triple of 3 multiplied by 5, which is 45, I set under the former summe 135, a place forwarder, and my cubick divisor 5 under the last summe a place forwarder, that is, under the third cubick point, these drawn into one summe will be 13955, and being substracted from the former summe 500000432915, the remainer 498.60493. 2915 is my divisor corrected, and yet I know not whether I have a true quotient or not; under this remainer therefore I draw a line, and work with 4, which I suppose to be the true quotient in manner following; and that the manner of the work may be the more perspicuous, (as in the trisection of an angle, so here) 3 the first figure found I call (*a*) and 4 the second figure I call (*e*) the square of three I note

with

with *aa*, the cube with *aaa*, the quadrato quadrat with *aaaa*, the quadrato cube with *aaaaa*, so likewise the square of 4 the second figure I note with *ee*, the cube with *eee*, the quadrato quadrate with *eeee*, the quadrato cube with *eeeee*; my first divisor I note with *ffff*, because this Equation is quadrato quadratick, and 5 my second divisor, I note with *ee*, because the divisor it self is cubick: these things premised, I proceed thus: First, I multiply 405, which is 5 *aaaa* or 5 times the quadrato quadrate of 3 by *e*, that is, by 4, and the product thereof 1620, I set under my divisor corrected, so as the last figure thereof may stand under the first figure of the third quadrato cubick number, and against this number I put in the margine 5 *aaaae*, that is, five times the quadrato quadrate of 3 multiplied by 4: next 270, ten times the cube of 3, by 16 the square of 4, and this product 4320, I set under the former a place forwarder, and 90, which is 10 times the square of 3, I multiply by 64, the cube of 4, & this product 5760 I set under the last a place forwarder then that, and 15, which is 5 times 3, I multiply by 256, the quadrato quadrate of 4, & the product thereof 3840, I set under the third product a place forwarder, and 1024, the

the quadrato cube of four under that; lastly, I multiply four, the last figure placed in the quotient by 5 my divisor, and the last figure of this product I set under 5 my divisor, and supposing ciphers to be thereunto annexed, I collect these several products into one summe, and their aggregate 200000211,35424, is five roots more one quadrato quadrate, under which I draw a line, and seek the five cubes to be substracted, thus.

First, I multiply 135 (which is thrice the square of three multiplied by five my cubick divisor) by four, the figure last placed in the quotient, and the product thereof 540 I set under the last summe, so as the last figure thereof may be under the first figure of the third cube; next I multiply 45, that is, five times the triple of three, by 16 the square of four, and this product 720 I set under the former a place forwarder, and under that 320, which is five times the cube of 4, a place forwarder too, these products drawn into one summe do make 61520, the five cubes to be substracted from the five roots more one quadrato quadrate before found, which being done, the remainer will be 199385081135424, and this remainer being substracted from the figures

figures of the ſubtenſe given over the head thereof, the remainer will be 450.79600.59892, and becauſe ſuch a ſubſtraction may be conveniently made, I conclude, that I have found the true quotient, and ſo have I wrought twice.

The work following muſt be done in all things like as this ſecond, onely remember that as in the triſection of an angle, both the figures in the quotient are termed *a* in the third operation, the three figures found are *a* in the fourth work; and ſo forward till your diviſion be finiſhed.

c	17431	14854	0 3
	5		*ffff*
		5	--*c c*
		243	+ *aaaaa*
	15000		+ *ff ffa*
	15000	00243	
	13	5	-- *ccaaa*
	14986	50243	*Subſtract*
Reſt	02444	64611	

Rest	02444	64611	95316	34711	ae 0,034.904
	5				ffff
		40	5....		5aaaa
		2	70..		10aaa
			90.		10aa
			15.		5a
	..500	00043	2915.		
	1	35..			--ce 3aa
		45.			--ce 3a
		5			--ce
	--1	3955			

498	60493	2915•		Divisor
	162	0••••		
	43	20•••		
	15	760••		
		3840•		
		1024		
20				
2000	00211	35484		
5	40•••			–cc3aaa
	720•.			–cc3aaa
	320			–ccccc
–6	1520			

Rest	1902	850000	35424		Subtract
	450	79000	50892	34711	-ccccc
					-cc [illegible]
	5	2 40 ..			ffff -cc [illegible]
		000	0068168	04 ..	5aaaa
		30	3930	40 ..	10aaa
			11	560 .	10aa
				170	5a
	50	00006	72198	9770	
		173	40.50 ..		
			5100 ..		
		908	0003	5612 .	Divisor

		173	91050	
49	99832	81059	9770	*Divisor.*
	60	13512	0...	5aaaae
	3	18362	40..	10aaaee
		8427	240.	10aaeee
		111	5370.	5aeeee
			59049	eeeee
450				ffffe
450	00065	40413	76749	
	56060			-ee3aae
48	4131	6.		-ee3aee
	02536	45		-ccbee

2	60227	33		
948	39835	95413	76749	Subſtract
Reſt	39764	64478	57962	
	5			ſſſſ

34 The Sines of two arches equally distant on both sides from 60 degrees, being given, to finde the Sine of the distance.

The Rule.

Take the difference of the Sines given, and that difference shall be the Sine of the arch sought.

The reason of the Rule.

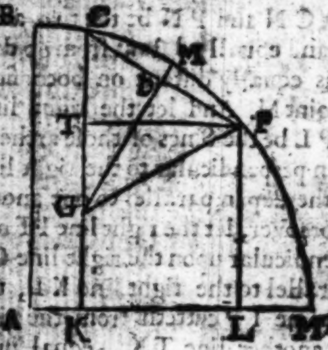
Let C N and P N be the two arches given, and equally distant from 60 deg. M N, that is equally distant on both sides from the point M. And let the right lines C K and P L be the Sines of those arches, being drawn perpendicular to the right line A N, and thereupon parallel to one another.

Moreover, let the right line PT be drawn perpendicular upon the right line C K, and so parallel to the right line K L, then this right line T P cutteth from the right line C K another line T K, equal unto P L, by the 15 of the second, and leaveth the right line T C for the difference of the Sines C K and P L. Lastly, the Sines of the distance of either of them from 60 degrees let be the right line C D or D P, I say, that the right line T C is equal to the right line C D or D P.

Demonstration.

Because in the triangle T C P, that the

perpendicular G D doth bisect the base C P, by the proposition: therefore the sides G C and G P are equall, and the angles G C P and G P C are equal, because equal sides subtend equal angles: and lastly, the angles C G D and D G P are also equal, by the same reason; but the angle C G D



is 30 degrees, for that it is equal to the angle B A M, because a right line drawn through two parallel right lines maketh the angles opposite to one another equall. And therefore the angle C G P is 60 degrees, because it is double to the angle C G D. And because the angle C G P is 60 degrees, therefore the other two angles G C P and G P C are 120, by the 18th. of

the

the second, and these two angles are demonstrated to be equall; and therefore every of them is 60 degrees. And the angle C G P is also 60 degrees, and therefore the triangle C G P is equiangled, but because the triangle C G P is equiangled, therefore also it is equilateral. Moreover, because the triangle C G P is equilateral, therefore the perpendicular P. K bisecteth the base C G into two equall parts, or else it could not be perpendicular. Then the sides C P and C G are equall, and therefore also their bi-segments C T and C D are equal; which was to be demonstrated. The Sines therefore of whatsoever 60 degrees being given, you may find the Sines of the other 30 degrees, by Addition or Subtraction onely.

Example.

Let the arches C N be 70 degrees, P N 50, C M or P M 10 degrees; for so many degrees are the arches of 70 degrees; and 50 degrees distant from the arch of 60 degrees on both sides. And let first the Sines of 70 degrees and 10 degrees be given, and let the Sine of 50 degrees be demanded.

From the Sine of 70d. CK 9396926
Substract the Sine of 10d. CD or CT, 1736482

The Remainer will be the Sine of 50d. TK or PL, 7660444

Then let the Sine of 70 degrees and 50 degrees be given, and let the Sine of ten degrees be demanded.

From the Sine of 70 degrees, CK, 9396926
Substract the Sine of 50d. TK or PL, 7660444

Remainer is the Sine of 10d. CD, 1736482

Lastly, let the Sines of 50 degrees and 10 degrees be given, and let the Sine of 70 degrees be demanded.

To the Sine of 50d. PL or TK, 7660444
Adde the Sine of 10d. DR or TC, 1736482

Their sum will be the Sine of 70d. 9396926

And thus far of the making of the Tables of right Sines, the Tables of versed Sines are not necessary, as hath been said.

CHAP.

CHAP. IV.

By the Tables of Sines to make the Tables of Tangents and Secants.

1. AS the Sine of the complement, Is to the Sine of an arch: ſo is the Radius, to the tangent of that arch.

2. As the Sine of the complement, is to the Radius; ſo is the Radius, to the ſecant of that arch. For, by the 16*th.* of the ſecond:

1. As the Sine of the complement A B, is to the Sine C A: ſo is the Radius B D or B C, to D E, the tangent.

2. As the ſine of the complement A B, is to the Radius D B or B C: ſo is the Radius B C, to the ſecant B F.

Example.

Let the tangent and ſecant of the arch C D 30 degrees be ſought for. The ſine of C A 30 degrees is 5000000, the ſine of the com-

complement A B 60 degrees is 8660254. Now then if you multiply the sine A C 5000000, by the Radius C B 10000000, the product wil be 50000000000000, which divided by the sine of the complement A B 8660254; the quotient will be 5773503, the right line F D of the tangent of the arch of 30 degrees.

2. As the sine of the complement A B 8660254, Is to the Radius D B 10000000: so is the Radius B C 10000000, to F B, the secant of the arch of 30 degrees: and so for any other; but with more ease by the help of these Theorems following.

Theor.

Theorem 1.

The difference of the Tangents of any two arches making a Quadrant, is double to the tangent of the difference of those arches.

The Declaration.

Let the two arches making a Quadrant be C D and B D, whose tangents are C G, and B P, and let B S be an arch made equall to C D; and then S D will be the arch of the difference of the two given arches C D or B S, and B D. And also let the tangent B T be equal to the tangent C G, and then the right line T P will be the difference of the tangents given C G or B T, and B P. Lastly, let the arches B L and B O (whose tangents are B K and B M) be made equal to the arch S D; I say, the right line T P being the difference of the two given tangents, C G and B P is double to the right line B K, being the tangent of the difference of the two given arches; or which is all one, I say, that the right line T P is equal to the right line M K.

Demonstration.

If you take equall things from equall, the remainders shall be equall; but the right angles

lines K P and M T are equal; therefore if you take the right line K T from both of them, the right lines T P and M K remaining shall be equal; because those things that are equal to one and the same things are also equal to one another; but the right lines K P and M T are equal to the

same right line K A, and therefore they are equal to one another. The right line K P is equal to the right line K A, because the angles K A P and K P A are equal. And that

that the angles K A P and K P A are equal to one another, thus appeareth; for that they are equal to one and the same angle D A C. The angle K P A is equal to the angle D A C, because the right line P A is drawn through the parallel lines M P and A C: and the angle K A P is equal to the angle D A C, by the construction, for the arch B L is to be made equal to the arch S D, being the difference of the arches D C and B D. Therefore the angle B A L or K A K is the difference betwixt the angles B A P and D A C. Seeing therefore that the angles K A P and K P A are equal to the same angle D A C; it followeth necessarily, that they are equal to one another.

Then that the right line M T is equall to the right line K A is thus proved; the right line M A is equall to the right line K A, by the work, but the right line M T is equal to the right line M A, and therefore it is also equall to the right line K A.

That the right line M T is equal to the right line M A doth thus appear; for that the angles M A T and M T A are equall; and therefore the sides opposite unto them are equal, for equall sides subtend equall angles: and the angles M T A and M A T are

are equal, because the angle M T A is equal to the angle T A C, by the like reason, that the angle K P A is equal to the angle D A C; and the angle M A T is equall to the angle T A C, by the proposition: for the arches C S and S O are put to be equal: therefore it followes, that they are also equal one to another. Generally therefore, the difference of the tangents of two arches, making a Quadrant, is double to the tangent of the difference of those arches, which was to be demonstrated. And by consequence, the tangents of two arches being given, making a Quadrant, the tangent of the difference of those arches is also given. And contrarily, the tangent of the difference of those two arches being given, together with the tangent of one of the arches, the tangent of the other arch is also given.

Example.

Let there be given the Tang. of [illegible] deg. [illegible] m.
And the Tang. of its complement, that is, of [illegible]
Halfe the difference of these two arches is [illegible]

Tangent of 72 de. 94 m. is 32586458
Tangent of 17 6 3068761
Their difference is 29517697
The halfe whereof is 14758848
The Tangent of 55 de. 88 min.
Or let the tangent of the greater arch 72 d. 94 m. be given, with the Tangent of the difference 55 de. 88 m. and let the lesser arch 17 de. 6 m. be demanded.
Tangent of 72 de. 94 m. is 32586458
Tang. of 55 de. 88 m. doubled is 29517676

Their difference is 3068782
The Tangent of 17 de. 6 m.
Or lastly, let the lesser arch be given, with the Tangent of the difference, and let the greater arch be demanded.
Tang. of 55 de. 88 m. the diff. is 14758838

Which doubled is 29517676
To which the tang. of 17 d. 6 m. ad. 3068761
Their aggregate is 32586437
the tangent of 72 degrees, 94 minutes.

Theor. 25.

The tangent of the difference of two arches within a Quadrant, with the tangent of the lesser arch maketh the secant of the difference.

Because the tangent of the difference BI

[illegible] Q; that is, the right line B K, or B M with the tangent of the lesser arch B S, that is, with the right line B T, maketh the right line M T, which is equall to the Secant A K, by the demonstration of the first Theoreme. Therefore, the tangent of the difference of two arches making a Quadrant, and the tangent of the lesser arch being given, the secant of the difference is also given. And contrarily.

For example,

Let the tangent of the former difference 31 degrees, 48 minutes; and the tangent of the lesser arch 29 degrees, 6 minutes, be given; to say, the secant of that difference is equall [illegible];

Tang. of the differ. 31 de. 48 m. is [illegible]

The tangent of 29 de. 6 m. is [illegible]

Their sum is the secant of 31 [illegible]

The tangent of the difference of two arches making a Quadrant, with the secant of their difference, is equall to the tangent of the greater arch.

Because the tangent of the arch B L, being the difference of the two arches B C and D C, making a Quadrant with the secant of the same arch B L, that is, the right

right line B K with the right line A K, is equal to the right line B P, by the demonstration of the first Theoreme: therefore the tangent of the difference of two arches making a Quadrant being given, with the secant of their difference, the tangent of the greater arch is also given.

For example.

Let the tangent of the difference be the tang. of the arch of 55 de. 30 m. viz. 145[illegible]
The secant of this difference is 1761[illegible]
Their sum is the tang. of 72 3/4 [illegible] the greater of the two former given arches.

And now by the like reason these Rules may be added by way of *Appendix*.

Rule 1.

The double tangent of an arch, with the tangent of half the complement, is equall to the tangent of the arch, composed of the arch given and half the complement thereof.

For if the arch B L be put for the arch given, the double tangent thereof shall be T P, by the demonstration of the first Theorem. And the complement of the arch B L, shall be the arch L C, whose half is the arch L D or D C, whose tangent is the right line G C or B T, but T P added to B T maketh B P, being the tangent of the arch

arch B D, composed of the given arch B L, and half the complement L D, therefore the double tangent, &c.

Rule II.

The tangent of an arch with the tangent of half the complement is equal to the secant of that arch. For if you have the arch B L or B O for the arch given, the tangent of the arch given shall be B M, the tangent of half the complement shall be B T, which two tangents added together, make the right line M T, but the right line M T is equal to the right line A K, by the demonstration of the first Theorem; which right line A K is the secant of the arch given B L, by the proposition: Therefore the tangent of an arch, &c.

Rule III.

The tangent of an arch with the secant thereof is equal to the tangent of an arch, composed of the arch given, and half the complement. For if you have the arch B L for the arch given, B K shall be the tangent, and A K the secant of that arch, But the right line A K and K P are equal, by the demonstration of the first Theorem; therefore the tangent of the arch given B L, that is, the right line B K, with the secant of the same arch, that is, A K is equall to the

the right line B P, which is the tangent of the arch B D, being composed of the given arch, B L and L D being half the complement.

These rules are sufficient for the making of the Tables of natural Sines, Tangents, & Secants, The use whereof in the resolution of plain & spherical triangles should now follow; but because the Right Honourable *John* Lord *Nepeir*, Baron of *Marchiston*, hath taught us how by borrowed numbers, called Logarithmes, to perform the same after a more easie and compendious way; we will first speak something of the nature and construction of those numbers, called Logarithmes; by which is made the Table of the artificial Sines and Tangents, and then shew the use of both.

CHAP. V.

Of the nature and construction of Logarithmes.

Logarithmes are borrowed numbers, which differ amongst themselves by Arithmetical proportion, as the numbers that borrow them differ by Geometrical proportion:

proportion: So in the first column of the ensuing Table the numbers Geometrically proportional being 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, &c. you may assigne unto them for borrowed numbers or Logarithmes, the numbers subscribed under the letters A, B, C, D, or any other at pleasure; provided, that the Logarithmes so assigned still differ amongst themselves by Arithmetical proportion, as the numbers of the first column differ by Geometrical proportion. For example, In the column C, if you will appoint 5 to be the Logarithme of one, 8 the Logarithme of 2, and 11 the Logarithme of 4, 14 must needs be the Logarithme of 8, the next proportional, because the numbers 5, 8, 11, and 14 differ amongst themselves by Arithmetical proportion, as 1, 2, 4, and 8 (the proportional numbers unto which they are respectively assigned) differ by Geometrical proportion, that is, as the numbers 5, 8, 11, and 14 have equal differences: so the numbers 1, 2, 4, and 8 have their differences of the same kinde: for as the difference between 5 and 8, 8 and 11, 11 and 14, is 3: so in the other numbers, as 1 is half 2, so 2 is half 4, 4 half 8, &c. The same observation may be made of the Logarithmes placed in

the

the columns A, B, and D, or of any other numbers which you ſhall aſſigne as Logarithmes unto any rank of numbers, which are Geometrically proportionall, and theſe Logarithmes or borrowed numbers you may propound to increaſe, and to be continued upwards, as thoſe of the columnes A, B, C, or otherwiſe to decreaſe, and to be continued downwards, as thoſe of the column D.

	A	B	C	D
1	1	5	5	35
2	2	6	8	32
4	3	7	11	29
8	4	8	14	26
16	5	9	17	23
32	6	10	20	20
64	7	11	23	17
128	8	12	26	14
256	9	13	29	11
512	10	14	32	8
1024	11	15	35	5
	Log	Log	Log	Log

The

The numbers continually proportional, which Mr. *Briggs* (after a conference had with the Lord *Nepeir*) hath proposed to himself in the Calculation of his *Chiliades*, are 1, 10, 100, 1000, &c. to which numbers he hath assigned for Logarithmes 0.00, &c. 1000, and 2000, and 3000, that is to say, to 1, the Logarithme 0.000, and to, 10, the Logarithme 1,000, and to 100 the Logarithme 2,000, as in the table following you may perceive. In the column marked by the letter A, there is a rank of numbers continually proportional from 1, and over against each number his respective Logarithme in the other column, signed by the letter B.

A	B
1	0.00000
10	1.00000
100	2.00000
1000	3.00000
10000	4.00000

Having thus assigned the Logarithme to the proportional numbers of 1, 10, 100, 1000, &c. in the next place, it is requisite to

to finde the Logarithmes of the mean numbers situate amongst those proportionals of the same table, viz. of 2, 3, 4, &c. which are numbers scituate betwixt 1 and 10; or 11, 12, 13, &c. which are placed betwixt 10 and 100; and so consequently of the rest; wherefore how this also may be done we intend to explain by that which followeth.

1. §. Make choice of one of the proportional numbers in the Table AB, and by a continued extraction of the square root create a rank of continuall meanes betwixt that number and 1, in such sort, that the continuall mean which cometh nearest 1 may be a mixt number, lesse then 2, and so near 1, that it may have as many ciphers before the significant figures of the numerator, as you intend that the Logarithmes of your Table shall consist of places.

Example.

In the premised Table AB, I take 10, the second proportional of that Table, then annexing unto it a compent company of ciphers, as twenty and four, thirty and six, fourty and eight, or any other number at pleasure, onely observe, that the more ciphers you annex unto the number given, the more just and exact the operation will prove; to make the Logarithmes of a

Table to seven places 28 ciphers will be sufficient, they being therefore added to 10, I extract the square root thereof, and finde it to be 3.16227766016837; again, annexing unto this root thus found 14 ciphers more, and working by that entire number so ordered, as if it were a whole number, I extract the root thereof, which I finde to be 1.77827941003892: and so proceeding successively by a continued extraction, I produce 27 square roots, or continual means betwixt 10 and 1, and write them down in the first column of the Table hereunto annexed, in which you may observe, that the three last numbers marked by the letters G, H, and I, viz.

1.00000006862238
1.00000003431119
1.00000001715559

are each of them mixt numbers lesse then 2, and greater then 1, and likewise to have seven ciphers placed before the significant figures of their numerators, according to the true meaning and intention of this present rule.

3. 6. Having thus pro[illegible] a great company of continual meanes, annex unto them their proper Logarithmes, by halfing first the Logarithme of the number taken

ken

ken, and then successively the Logarithme of the rest.

For example.

1.0000000000000000 being assigned the Logarithme of 10, the number taken 0.500000, &c. marked by the letter D, in the second column of the following Table, which is the half of 1.0000, &c. is the Logarithme of the number A, the square root of 10: in like manner 0.25000, &c. being half 0.5000, and is the Logarithme of the number B, and 0.125000, &c. is the Logarithme of the number C, and so of the rest in their order. So that at last, as you have in the first column of the following Table 27 continuall meanes, betwixt 10 and 1, as aforesaid: So in the other column you have to each of those continuall meanes, his respective Logarihme.

3. §. When a number which being lesse then 2, and greater then 1, comes so neer to 1, that it hath seven ciphers placed before the significant figures of the numerator, the first seven significant figures of the numerator of such a number, and the first seven significant figures of the numerator of his square root lessen themselves like their Logarithmes, that is, by halfes.

This is proved by the Table following;

 for

for there in the second column thereof, the number N being the Logarithme of the number G, I say, as the Logarithme K is half the Logarithme N, so 3431119, the first seven figures of the numerator of the number H, are half 6862238, the first seven significant figures of the numerator of the number G. Any two numbers of this kinde therefore being given, their Logarithmes and the significant figures of their numerators are proportional.

Example. The numerators G and H being given, I say, as 6862238, the significant figures of the numerator of the number G, are to 3431119, the significant figures of the numerator of the number H; so is 29802322, the Logarithme of the number G, to 140901161, the Logarithme of the number H. In like manner, G and L being given, as 6862238, is to 1715559, so is 29802322, the Logarithme of the number G, to 7450580, the Logarithme of the number L. This holdeth also true in any other number of this kinde, though it be not one of the continual means betwixt 10 and 1, for the significant figures of the numerator of any such number bear the same proportion to his proper Logarithme, that the significant figures of any of the numbers marked by the letters G, H, or L bear to his.

	10.0000, &c.	1.000000000000000	
A	3.16227766016837	0.500000000000000	D
B	1.77827941003892	0.250000000000000	
C	1.33352143216332	0.125000000000000	
	1.15478198468945	0.062500000000000	
	1.07460782832131	0.031250000000000	
	1.03663292843769	0.015625000000000	
	1.01815172171818	0.007812500000000	
	1.00903504484144	0.003906250000000	
	1.00450736425446	0.001953125000000	
	1.00225114829291	0.000976562500000	
	1.00112494139987	0.000488281250000	
	1.00056231260220	0.000244140625000	
	1.00028111678778	0.000122070312500	
	1.00014054851694	0.000061035156250	
	1.00007027178941	0.000030517578125	
	1.00003513527746	0.000015258789062	
	1.00001756748442	0.000007629394531	
	1.00000878270363	0.000003814697265	
	1.00000439184217	0.000001907348632	
	1.00000219591867	0.000000953674316	
	1.00000109795873	0.000000476837158	
	1.00000054897921	0.000000238418579	
	1.00000027448957	0.000000119209289	
	1.00000013724477	0.000000059604644	
G	1.00000006862238	0.000000029802322	N
H	1.00000003431119	0.000000014901161	K
L	1.00000001715559	0.000000007450580	M

4. §. These things being thus cleared, it is manifest, that a number of this kinde being given, the Logarithme thereof may be found by the Rule of three direct. For as the significant figures of the numerator of any one of the numbers (signed in the first column of the last Table by the letters G, H, or L.) are to his respective Logarithme: so are the significant figure of the numerator of the number given, to the Logarithme of the same number.

Example. The number 1.0000000010213o1 being given, I demand the Logarithme thereof: I say then,

As 6862238, the significant figures of the numerator of the number G, are to 2980-2322, the logarithme of the same number G: so are 1021301, the significant figures of the numerator of the number given, to 4357281, the Logarithme sought; before which if you prefix 9 ciphers, to the intent it may have as many places as the Logarithme in the last premised Table, (*viz.* 16) the true and entire Logarithme of 1.00000-001021301, the number given is 0,000000-004357281, as before. And to every Logarithme thus found, you must prefix as many ciphers as will make the said Logarithme to have as many places as the other Loga-

Logarithmes in the ſame table: for though you make your Table of Logarithmes to conſiſt of as many places as you pleaſe, yet when you are once reſolved of how many places the Logarithmes of your Table ſhall conſiſt, you muſt not alter your firſt reſolution, as to make the Logarithme of 2 to conſiſt of ſix places, and the Logarithme of 16 to have ſeven, but if the ſignificant figures of the numerator of the Logarithme of 2 have not ſo many places as the ſignificant figures of the Logarithme of 16, you muſt prefix a cipher or ciphers to make them equal; becauſe (as hath been ſaid, the Logarithmes of this kinde ought all to conſiſt of equal places in the ſame Table.

5. §. Now then to finde the Logarithme of any number whatſoever, you are firſt to ſearch out ſo many continual means betwixt the ſame number and 1, till the continual mean that cometh neereſt 1 hath as many ciphers placed before the ſignificant figures of his numerator, as you intend the Logarithmes of your Table ſhall conſiſt of places; Again, this being done, you are to finde the Logarithme of that continual mean: And laſtly, by often doubling and redoubling of that Logarithme ſo

found (according to the number of the continual meanes produced) in conclusion you shall fall upon the Logarithme of the number given.

Example. the number 2 being given, I demand the Logarithme thereof to seven places: Here first in imitation of that which is before taught in the first rule of this Chapter, I produce so many continual meanes between 2 and 1, till that which cometh nearest 1 hath seven ciphers before the significant figures of the numerator, which after three and twenty continued extractions, I finde to be 1.00000008262958. This continual mean being thus found (by the direction of the last rule aforegoing) I finde the Logarithme thereof to be 0.000000035885571. for,

As 6862238, is to 29802322:
So 8262958, is to 35885571.

This Logarithme being doubled will produce the Logarithme of the continual mean next above 1.00000008262958, and so by doubling successively the Logarithme of each continual mean one after another, according to the number of the extractions (*viz.* three and twenty times in all) at last you shall happen upon the Logarithme 0.30102998797168, which is the Loga-

rithme

rithme of 2 the number propounded: The whole frame of the work is plainly set down in the table following; for in the first column thereof you have 23 continual meanes betwixt 2 and 1, and in the other column their respective Logarithmes, found by a continual doubling and redoubling of 0,000000035885571, the Logarithme of the last continual mean in the table.

2.0000, &c.	0 301029987975168
1. 41421356237309	0.150514993987584
1. 18920711500272	0.075257496993792
1. 19050713266525	0.037628748496896
1. 04427378243220	0.018814374248448
1. 02189714865645	0 009407187124224
1. 01088928605285	0.004703593562112
1. 00542990111387	0.002351796781056
1. 00271127505073	0.001175898390528
1. 00135471989237	0.000587949195264
1. 00067713069319	0.000293974597632
1. 00033850805274	0.000146987298816
1. 00016923970533	0.000073493649408
1. 00008461627271	0.000036746824704
1. 00004230724140	0.000018373412352
1. 00002115339696	0.000009186706176
1. 00001057664255	0.000004593353088
1. 00000528830729	0.000002296676544
1. 00000264415015	0.000001148338272
1. 00000132207420	0.000000574169136
1. 00000066103688	0.000000287084568
1. 00000033051838	0.000000143542284
1. 00000016525917	0.000000071771142
1. 00000008262958	0.000000035885571

But

But now becauſe the Logarithme of the number propounded was to conſiſt onely of ſeven places ; therefore of the Logarithme ſo found I take onely the firſt ſeven figures rejecting the reſt as ſuperfluous, and then at the laſt the proper Logarithme of 2, the number given will be found to be 0.301029, and becauſe the eighth figure being 9, doth almoſt carry the value of an unit to the ſame ſeventh figure, I adde one thereto, and then the preciſe Logarithme of 2 will be 0.301030. And thus as the Logarithme of 2 is made, ſo may you likewiſe make the Logarithme of any other number whatſoever : Howbeit, the Logarithmes of ſome few of the prime numbers being thus diſcovered, the Logarithmes of many other derivative numbers may be found out afterwards without the trouble of ſo many continued extractions of the ſquare root, as ſhall appear by that which followes.

6. §. When of four numbers given, the ſecond exceeds the firſt as much as the fourth exceeds the third ; the ſumme of the firſt and fourth is equal to the ſumme of the ſecond and third ; and contrarily.

As 8, 5 : *6*, 3. here 8 exceeds 5, as much as *6* exceeds 3 ; therefore the ſumme of

the first and fourth, namely, of 8 and 3 is equall to the summe of the second and third, namely of 5 and 6: from whence necessarily followes this *Corollary*;

When four numbers are proportionall, the summe of the Logarithmes of the mean numbers is equal to the summe of the Logarithmes of the extreams.

Example.

Let the four proportional numbers be those exprest in the first column of the first Table in this Chapter, *viz.* 4, 16, 32, 128, in which Table the Logarithme of 4 under the letter A is 3, the Logarithme of 16, 5, the Logarithme of 32, 6; and the Logarithme of 128 is 8. Now as the summe of 5 and 6, the Logarithmes of the mean numbers do make 11, so the summe of 3 and 8, the Logarithmes of the extreames, do make 11 also.

7. §. When four numbers be proportional, the Logarithmes of the first substracted from the summe of the Logarithmes of the second and third, leaveth the Logarithme of the fourth.

Example.

Let the proportion be, as 128, to 32; so is 16, to a fourth number: here adding 5 and 6, the Logarithmes of the second and third,

third, the sum is 11, from which substracting 8, the Logarithme of 128, the first proportional, the remainer is 3, the Logarithm of 4, the fourth proportional.

8. §. If instead of substracting the aforesaid Logarithme of the first, we adde his complement arithmetical to any number, the totall abating that number, is as much as the remainer would have been.

The complement arithmetical of one number to another, (as here we take it) is that, which makes that first number equall to the other; thus the complement arithmetical of 8 to 10 is 2, because 8 and 2 are 10. Now then whereas in the example of the last Proposition, substracting 8 from 11, there remained 3, if instead of substracting 8, we adde his complement arithmeticall to 10, which is 2, the totall is 13, from which abating 10, there remains 3, as before: both the operations stand thus:

As 128,	Logar	8 compl. arithmetical	2.
is to 32;		6	6.
So is 16,		5	5
The aggreg. of 1.2.		11 Their aggregate is	13.
To 4,		3	

from which abate 10, there remaines 3, and the like is to be understood of any other.

The

The reason is manifest, for whereas we should have abated 8 out of 11, we did not onely not abate it, but added moreover his complement to 10, which is 2; wherefore the total is more then it should be by 8 & 2, that is by 10; wherefore abating 10 from it, we have the Logarithme desired; which rule, although it be generall, yet we shall seldome have occasion to use any other complements, then such as are the complements of the Logarithmes given either to 10.000000, or to 20.000000, the & complement arithmetical of any Logarithme to either of these numbers, is that which makes the Logarithme given equal to either of them. Thus the complement arithmetical of the Logarithme of 2 *viz.* 0301030, is 9698970, because these two numbers added together, do make 10.000000, and thus the complement thereof to 20.000000 is 19698970 : if therefore 0301030 be substracted from 10.000000, the remainer is his complement arithmetical.

But to finde it readily, you may instead of substracting the Logarithme given from 10.000000, write the complement of every figure thereof unto 9, beginning with the first figure toward the left hand, and so on, till you come to the last figure towards the right

right-hand, and thereof set down the residue unto 10: Thus for the complement arithmetical of the aforesaid Logarithme, 0301030; I write for 0, 9: for 3, 6: for 0, 9: for 1, 8: for 0, 9: for 3 again I should write 6: but because the last place of the Logarithme is a cipher, and that I must write the complement thereof to 10, instead of 6 I write 7, and for 0, 0: and so have I this number, 5698970, which is the complement arithmetical of 0301030, as before.

9. §. Every Logarithme hath his proper Characteristick, and the Character or Characteristicall root of every Logarithme is the first figure or figures towards the left hand, distinguished from the rest by a point or comma. Thus the Character of the Logarithmes of every number lesse then 10 is 0, but the Character of the Logarithme of 10 is 1; and so of all other numbers to 100, but the Character of the Logarithme of 100 is 2; and so of the rest to 1000; and the Character of the Logarithme of 1000 is 3; and so of the rest to 10000: in brief, the Characteristick of any Logarithme must consist of a unite lesse then the given number consisteth of digits or places. And therefore by the Character of a

Loga-

Logarithme you may know of how many places the absolute number answering to that Logarithme doth consist.

10. §. If one number multiply another, the summe of their Logarithme is equal to the Logarithme of the product.

As let the two numbers multiplied together be 2, and 2 the product is 4, I say then that the summe of the Logarithmes of 2 and 2, or the Logarithme of 2 doubled is equal to the Logarithme of 4, as here you may see.

2.	0.301030
2.	0.301030
4.	0.602060

Again, let the two numbers multiplied together be 2, and 4, the product is 8, I say then, that the summe of the Logarithmes of 2 and 4 is equall to the Logarithme of 8, as here you may also see,

2.	0.301030
4.	0.602060
8.	0.903090.

And so for any other.

The reason is, for that (by the ground of multiplication) as unit is in proportion to the multiplier: so is the multiplicand, to the

the product: therefore (by the sixth of this Chapter) the sum of the Logarithmes of a unit, and of the product is equall to the summe of the Logarithmes of the multiplier and multiplicand, but the Logarithme of a unit is 0, therefore the Logarithme of the product alone is equal to the summe of the Logarithmes of the multiplier and multiplicand.

And by the like reason, if three or more numbers be multiplied together, the summe of all their Logarithmes is equall to the Logarithme of the product of them all.

11. §. If one number divide another, the Logarithme of the Divisor being substracted from the Logarithme of the Dividend, leaveth the Logarithme of the Quotient.

As let 10 be divided by 2, the quotient is 5. I say then, if the Logarithme of 2 be substracted from the Logarithme of 10, there will remain the Logarithme of 5, as here is to be seen.

10.	1.000000
2.	0.301030
5.	0.698970.

For seeing that the quotient multiplied by the divisor produceth the dividend, there-

therefore, by the last proposition, the sum of the Logarithmes of the quotient and of the divisor is equal to the Logarithme of the dividend: if therefore the Logarithme of the dividend, be substracted from the Logarithme of the divisor, there remaines the Logarithme of the quotient.

12. §. In any continued rank of numbers Geometrically proportionall from 1, the Logarithme of any one of them being divided by the denomination of the power which it challengeth in the same rank, the quotient will give you the Logarithme of the root. In the rank of the proportional numbers of the Table A B C D, 2 being the root, or first power; 4 the square or second power, 8 the cube, or third power, 16 the bi-quadrate or fourth, 32 the fifth power, 64 the sixth power, &c. I say, the Logarithme of 4, 8, 16, 32, 64, or of any of the other subsequent proportionals in that rank, being divided by the demonination of the power that the same proportional claimeth in the same rank, you shall finde in the quotient the Logarithme of 2 the root.

For example.

In the same Table the Logarithme of 4, the square or second power, *viz.* 3. being given

given, I demand the Logarithme of 2, the root: here the denomination of the power that the proportional 4 challengeth in that rank (being the square or second power) is 2, wherefore if 3, the Logarithme of 4 be divided by 2, the quotient will be 1, and there will remain 1 for a fraction; so that you see it cometh very near in the Logarithmes of but one figure, but if you take it to seven places, as in this table is intended, yon shall finde it exactly: for then the Logarithme of 4 will be 0.602060, and this being divided by 2, the quotient will be 0.301030, the Logarithme of 2 the root. So likewise 0.903090, the Logarithme of 8 the third power, being divided by 3, leaves 0.301030 in the quotient, as before, and so of any other.

13. §. In any rank of numbers Geometrically proportionall from 1, the Logarithme of the root being multiplied by the denomination of any of the powers, the product is the Logarithme of the same power.

This Rule is the inverse of the last.

For example.

In the rank produced in the last rule 0,301030, (the Logarithme of 2 the root) being doubled, or multiplied by 2, produceth

ceth 0.602060, the Logarithme of 4, the square or second power, and the same Logarithme of 0.301030, being trebled or multiplied by 3, produceth 0.903090, the Logarithme of 8, the cube or third power, and so of the rest.

The truth of these two last rules may thus be proved. In arithmeticall proportion, when the first term is the common difference of the terms, the last term being divided by the number of the terms, the quotient will give you the first term of the ranke: again, in this case, the first term multiplied by the number of the termes produceth the last term. So this rank 3, 6, 9, 12, 15, 18, 21 being propounded, wherein three is both the first term and also the common difference of the terms: I say, 21, the last term being divided by 7, the number of the termes, the quotient is 3, the first term. Contrariwise, 3 the first term multiplied by 7, produceth 21, the last term; and by the like reason, 0. 301030 being the first term, and also the common difference of the termes, that is, of the Logarithmes of 4, 8, 16, 32 and 64, the Logarithme of 2 the first term, being multiplied by 6, the number of the terms, produceth the Logarithme of 64, the last term, and the

Lo-

garithme of 64, the last term, being divided by 6, leaveth in the quotient the Logarithme of 2 the root.

Hence it also followes, that if you adde the Logarithme of 2, the common difference of the termes, to the Logarithme of any term, their aggregate shall be the Logarithme of the next term. Thus if I adde 0.301030, the Logarithme of 2 the root or first term, to 0.903090, the Logarithme of 8, the third term, their aggregate is 1.204120, the Logarithme of 16, the fourth term; and so of the rest.

14. §. Thus having shewed the constru-ction of the Logarithmetical Tables, the converting of the Table of natural Sines, Tangents and Secants into artificiall cannot be difficult, the artificiall Sines and Tangents being nothing but the Logarithmes of the naturall.

15. §. In the conversion whereof Mr. *Briggs* in his *Trigonometria Britannica*, thought fit to make the Radius of his natural Canon to consist of 16 places, and to confine his artificiall to the Radius of eleven, whose Characteristick is 10, but the Characteristick of the rest of the Sines till you come to the sine of 5 degrees and 73 centesmes is 9, and from thence to 57

cen-

centesimes, the Characteristick is 8, and from thence 7, till you come to 5 centensimes, and from thence but 6, to the beginning of the Canon. The Characteristick still decreasing in the same proportion with the naturall numbers, and the number of the places in the naturall Canon, do therefore exceed the Characteristick in the artificiall, that so the artificiall numbers might be the more exact.

16. §. In the Canon herewith printed, the Characterist.ck in the artificiall numbers doth exceed the number of places in the naturall, which is not done so much out of neceffity as conveniency, for the artificiall numbers in this Canon might in all respects have been made answerable to the natural, and so the Characteristick of the Radius, or whole Sine would have been seven, the Characterick of the first minute 3, but thus the subduction of the Radius would not have been so ready as now it is, nor yet the Canon it self altogether so exact, and therefore as Master *Briggs* confined the Radius of his artificiall Canon to eleven places for conveniency sake, though he made the Logarithmes to the

the Radius of sixteen: so here for conveniency and exactnesse both, the same Characterick is here continued, though the naturall numbers do not require it, if any think this a defect, I answer, that it could not well be avoided here, but may be supplied by Master *Briggs* his Canon, of which this is an abbreviation: and yet even here there is so small a difference between the Logarithmes of these naturall numbers, and the Logarithmes in the Canon, that any one may well perceive the one to be nothing else but the Logarithme of the other, if they do but change the Characteristick.

And hence we may gather, that the making of this Canon is not so difficult as laborious, and the labour thereof may be much abridged by this Proposition following.

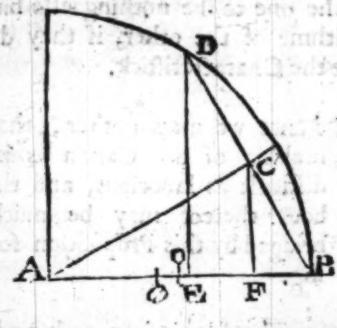
17. §. The Sine of an arch and half the Radius are mean proportionals between the Sine of halfe that arch, and the Sine complement of the same arch.

In the annexed Diagram, let DE be the

the ſine of 56 degrees, B C the ſine of 28, A C the ſine complement thereof, that is, of 62. D B the ſubtenſe of 56. C F perpendicular to the Radius, then are A B C and A C F like triangles, by the 22 of the ſecond, and their ſides proportional that is,

A B : B C :: A B : A O

A C : C F :: D E : C F



And therefore the oblongs of B C * A C, A O * D E, and A B * C F are equal, and the ſides of equal rectangled figures reciprocally proportional, that is, as BC, AO :: DE, AC, or as AO, BC :: AC, DE.

If therefore you multiply A O, the half Radius

Radius, by DE, the ſine of the arch given, and divide the product by BC, the ſine of half the arch given, the quotient ſhall be AC, the ſine complement of half the given arch.

Or if you multiply BC, the ſine of an arch by AC, the ſine complement of the ſame arch, and divide the product by AO, the half Radius, the quotient ſhall be DE, the ſine of the double arch. And therefore the ſines of 45 degrees being given, or the Logarithmes of thoſe ſines, the reſt may be found by the rule of proportion. For illuſtration ſake we will adde an example in naturall and artificiall numbers.

Naturall,

As BC 28,	46947
Is to AO 30;	50000
So is DE 56,	82903
To AC 62:	88294

Logarith.

As BC 28,	9,671609
Is to AO 30;	9.698970
So is DE 56,	9.918574
To AC 62.	9.945935

18. §. The compoſition of the naturall Tangents and Secants, by the firſt and ſecond

cond of the fourth are thus to be made.

1. As the ſine of the complement, is to the ſine of an arch: So is the Radius, to the tangent of that arch.

2. As the ſine of the complement, is to the Radius: ſo is the Radius, to the Secant of that arch; and by the ſame rules may be alſo made the artificiall; but with more eaſe, as by example it will appear.

Let the tangent of 30 degrees be ſought.

	Logarith.
As the co-ſine of 60 degrees;	9.937531
Is to the ſine of 30;	9.698970
So is the Radius,	10.000000
To the tangent of 30:	9.761439

And thus having made the artificiall Tangents of 45 degrees, the other 45 are but the arithmeticall complements of the former, taken as hath been ſhewed in the eighth rule of the fifth Chapter.

Again, let the ſecant of 30 degrees be ſought.

As the co-sine of 60 degrees,	9.937531
Is to the Radius,	10.000000
So is the Radius,	10.000000
	20.000000
To the secant of 30:	10.062469

And thus the Radius being added to the arithmetical complement of the sine of an arch, their aggregate is the secant of the complement of that arch. And this is sufficient for the construction of the naturall and artificiall Canon. How to finde the Sine, Tangent or Secant of any arch given in the Canon herewith printed, shall be shewen in the Preface thereunto: here followeth the use of the naturall and artificiall numbers both; first, in the resolving any Triangle, and then in *Astronomy*, *Dialling*, and *Navigation*.

CHAP. VI.

The use of the Tables of natural and artificial Sines, and Tangents, and the Table of Logarithmes.

In the Dimension

1. *Of plain right angled Triangles.*

THe measuring or resolving of Triangles is the finding out of the unknown sides or angles thereof by three things known, whether angles, or sides, or both; and this by the help of that precious gemme in Arithmetick, for the excellency thereof called the Golden Rule, (which teacheth of four numbers proportional one to another, any three of them being given, to finde out a fourth) and also of these Tables aforesaid.

Of Triangles, as hath been said, there are two sorts; plain and sphericall. A triangle

angle upon a plain is right lined, upon the Sphere circular. Right lined Triangles are right angled or oblique.

A right angled, right-lined Triangle we ſpeak of firſt, whoſe ſides then related to a circle are inſcribed totally or partially.

Totally, if the ſide ſubtending the right angle be made the Radius of a Circle, and then all the ſides are called Sines, as in the Triangle A B C.

Partially, if either of the ſides adjacent to the right angle be made the Radius of a circle, and then one ſide of the Triangle is the Radius or whole Sine, the ſhorter of

 the

the other two sides is a Tangent, and the longest a secant. Now according as the right angled Triangle is supposed, whether to be totally or but partially inscribed in a circle; so is the trouble of finding the parts unknown more or lesse, whether sides or angles; for if the triangle be supposed to be totally inscribed in a circle, we are in the solution thereof confined to the Table of Sines onely, because all the sides of such a triangle are sines: but if the triangle be supposed to be but partially inscribed in a circle, we are left at liberty to use the Table of Sines, Tangents, or Secants, as we shall finde to be most convenient for the work.

In a right angled plain Triangle, either all the angles with one side are given, and the other two sides are demanded, I say, all the angles, because one of the acute angles being given, the other is given also by consequence.

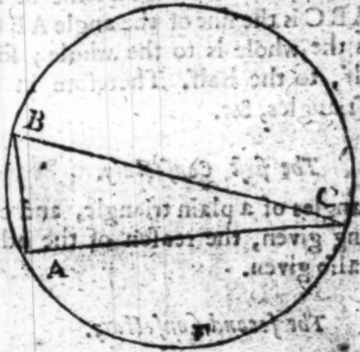
Or else two sides with one angle, that is, the right angle are given and the other two angles with the third side are demanded. In both which cases this *Axiome* following is well nigh sufficient.

The first AXIOME.

In all plain Triangles, the sides are in pro-

portion one to another, as are the sines of the angles opposite to those sides.

As in the triangle A B C, the side A B is in proportion to the side A C, as the sine of the angle at B is in proportion to the sine of the angle at C, and so of the rest.



Demonstration.

The circle ADF being circumscribed about the Triangle A B C, the side A B is made the chord or subtense of the angle A C B, that is, of the arch A B, which is opposite to the angle A C B. The side A C is made the subtense of the angle A B C; and the side B C is made the subtense of the angle

BAC, and are the double measures thereof, by the 19 Theorem of the second Chapter; therefore the side AB is in proportion to the side AC, as the subtense of the angle ACB is in proportion to the subtense of the angle ABC, but half the subtense of the angle ACB is the sine of the angle ACB, and half the subtense of the angle ABC is the sine of the angle ABC; now as the whole is to the whole; so is the half, to the half. Therefore in all plain Triangles, &c.

The first Consectary.

The angles of a plain triangle, and one side being given, the reason of the other sides is also given.

The second Consectary.

Two sides of a plain Triangle, with an angle opposite to one of them being given, the reason of the other angles is also given, by this proportion.

If the side of a Triangle be required, put the angle opposite to the given side in the first place.

If an angle be sought, put the side opposite to the given angle in the first place.

For

For the better understanding whereof we will adde an example, and to distinguish the sides of the Triangle, we call the side subtending the right angle, the Hypothenusall, and of the other two the one is called the perpendicular, and the other the base, at pleasure, but most commonly the shortest is called the perpendicular, and the longer the base. As in the former figure, the side B C is the Hypothenusal, A C the base, and A B the perpendicular.

Now then in the Triangle A B C, let there be given the base A C 768 paces, and the angle C B A 67 degrees, 23 minutes, (then the angle A C B is also known, it being the complement of the other) and let there be required the perpendicular: Because it is a side that is required, I put the angle opposite to the given side in the first place, and then the proportion is:

As the sine of the angle at the perpendicular, is in proportion to the base: So is the sine of the angle at the base, to the perpendicular.

Now if you work by the natural Sines, you must multiply the second term given, by the third, and divide the product by the

first, and then the quotient is the fourth term required, and the whole work will stand thus:

As sine the ang. at the perpend. A B C 67 degrees 40 minutes	9232102
Is in proportion to the base A C;	768
So is sine the angle at the base, A C B 22 degrees 60 minutes	3842953
	30743624
	23057718
	26900671
The product of the 2d. & 3d.	2951387904

Which divided by 9232102, the first term given, leaveth in the quotient 320 *feet*.

But if you work by the artificiall sines, that is, by the Logarithmes of the natural, then you must adde together the Logarithmes of the second and third terms; given, and from their aggregate substract the Logarithme of the first, and what remaineth will be the Logarithme of the fourth proportional, whether side or angle: the work standeth thus.

As

As sine the angle at the perpendicular B 67 deg. 40 min. } 9.9653006

Is in proportiō to the base AB 768; 2.8853612
So is sine the angle at the base C 22 degrees, 60 minutes. } 9.5846651

The aggregate of the 2d. & 3d. 12.4700263
From which I substract the first, 9.9653006,
and the remainer which is 2.5047251, is the Logarithme of the fourth: wherefore looking in the Table for the absolute number answering thereunto, I finde the nearest to be 320, which is the length of the perpendicular, as before.

The operation it self may yet be performed with more ease, if instead of the Logarithme of the first proportional, we take his complement arithmetical, as hath been shewed in the eighth rule of the fifth Chapter: for then the totall of the arithmeticall complement, and the Logarithme of the second and third proportionals, abating Radius, is the Logarithme of the fourth proportionall, as doth appear in this example.

As sine of ABC 67 de. 40 m. co. ar. 0.0346994
To the base A C 768; 2.8853612
So the sine of ACB 22 de. 60 m. 9.5846651

To the perpendic. AB 320 *feet* 2.5047257

Thus having sufficiently explained the operation in this first example, we shall be briefer in the rest that follow, understanding the like in them also.

In this manner may all the cases of a plain right angled Triangle be resolved by this proportion, except it be when the base and perpendicular with their contained angle (that is the right angle) is given, to finde either an angle or the third side; in this case therefore we must have recourse to the 17*th*. Theorem of the second Chapter, by help whereof the Hypothenusall may be found in this manner; square the sides, and from the aggregate of their squares extract the square root, that square root shall be the length of the Hypothenusal. *For example.* Let the base be four paces, and the perpendicular 3, the square of the base is 16, the square of the perpendicular is 9, the summe of these two squares is 25, the square root of this summe is 5 paces, and that is the length of the hypotenusal; and this hypothenusal being thus

thus found, the angles also may be found, as before.

Nor are we tied to this way of finding the hypothenusal, unlesse we confine our selves to the Tables of Sines onely; if we would make use of the Tables of Tangents or Secants, the hypothenusal may not onely be found with more ease, but all the cases of a right angled plain triangle may be also found severall wayes, by the help of this Axiome following.

The second AXIOME.

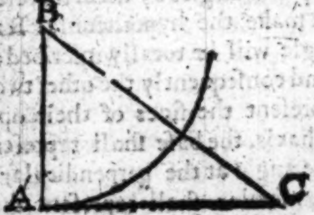
In a plain right angled triangle, any of the three sides may be made the Radius of a circle, and the other sides will be as Sines, Tangents, or Secants. And what proportion the side put as Radius hath unto Radius; the same proportion hath the other sides unto the Sines, Tangents, or Secants of the opposite angles by them represented.

If you make the hypothenusal Radius, the triangle will be totally inscribed in the circle, and consequently the other two sides shall represent the sines of their opposite angles, that is, the base shall represent the sine of the angle at the perpendicular, and the perpendicular shall represent the sine of the angle at the base, as in the preceding Diagram.

If

If you make the base Radius, the triangle will be but partially inscribed in the circle, and the other two sides shall be one of them a tangent, and the other a secant. Thus in the first Diagram of this Chapter, the base A D is made the Radius of the circle, the perpendicular D E is the tangent of the angle at the base, and A E is the secant of the same angle.

If you make the perpendicular Radius, the triangle will be but partially inscribed in the circle, as before, and the other two sides will be also the one a tangent and the other a secant. As in this example, the perpendicular A B is made the Radius of the circle, the base A C is the tangent of the angle at the perpendicular, and the hypothenusal B C is the secant of the same angle.



Hence

Hence it followes, that if you make A B the Radius, the base and perpendicular being given, the angle at the perpendicular may be found by this proportion.

As the perpendicular, is in proportion to Radius: So is the base, to tangent of the angle at the perpendicular; for the perpendicular being made the Radius of the circle, it must of necessity bear the same proportion unto Radius, as the hypothenusal doth, when that is made the Radius of the circle; and if the perpendicular be the Radius, the base must needs represent the tangent of the angle at the perpendicular.

And the angle at the perpendicular bebeing thus found, the hypothenusal may be found by the first Axiome. For,

As the sine of the angle at the perpendicular, is in proportion to his opposite side the base; So is Radius, to his opposite side the hypothenusal: and thus you see that the hypothenusal may be found without the trouble of squaring the sides, and thence extracting the square root. And hence also all the cases of a right angled plain triangle may be resolved several wayes: that is to say,

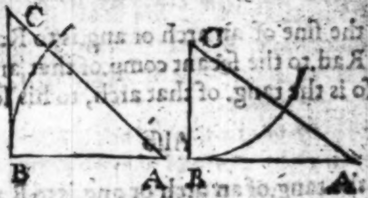
1. In a plain right angled triangle: the angles and one side being given, every of the

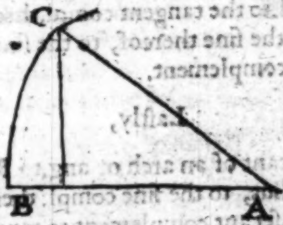
the other ſides is given, by a threefold proportion, that is, as you ſhall put for the Radius, either the ſide ſubtending the right angle, or the greater or leſſer ſide including the right angle.

2. Any of the two ſides being given, either of the acute angles is given by a double proportion, that is, as you ſhall put either this or that ſide for the Radius: to make this clear, we will firſt ſet down the grounds or reaſons for varying of the termes of proportion; and then the proportions themſelves in every caſe, according to all the variations.

The reaſons for varying of the termes of proportion are chiefly three.

The firſt reaſon is, becauſe the Radius of a circle doth bear a threefold proportion to a ſine, tangent, or ſecant; and contrariwiſe, a ſine, tangent, or ſecant hath a threefold proportion to Radius, by the ſecond Axiome of this Chapter.





For

As sine BC, to Rad. AC in the 1. triangle;
So Rad. BC, to secant AC in the 3d. tri.
So tang. BC, to secant AC in the 2d. tri.
& contra.

Again,

As tang. BC, to Rad. AB in the 2d. triang.
So Rad. BC, to tang. AB in the 3d. trian.
So sine BC, to sine BA in the first triang.
& contra.

Lastly,

As secant AC, to Rad. BC in the 3d. tri.
So Rad. AC, to sine BC in the first trian.
So secant AC, to tang. BC in the 2d. tri.
& contra.

Hence

Hence then

As the sine of an arch or ang. is to Rad. | & contr.
So Rad. to the secant comp. of that arch |
& so is the tang. of that arch, to his sec. |

Also

As the tang. of an arch or ang. is to Rad. | & contra.
So is Rad. to the tangent compl. thereof. |
And so is the sine thereof, to the sine of |
its complement. |

Lastly,

As the secant of an arch or ang. to Rad. | & contra.
So is Radius, to the sine compl. thereof |
And so is secant complement to tangent |
complement thereof. |

Example.

Let there be given the angle at the perpendicular 41 degrees 60 minutes, and the base 768 paces, to finde the perpendicular.

First, by the natural numbers,

As the secant of BAC 41 d. 60 m.	13372593
Is to Radius,	10000000
So is the base A B	768
To the perpendicular B C	574

By

By the Artificiall.

As the secant of BAC 41. 60.	10.1262157
Is to Radius ;	10.0000000
So is the base 768,	2.8853612
	12.8853612
To the perpendicular 574 :	2.7591455

Secondly, by the naturall numbers.

As the Radius,	10000000
To the co-sine of BAC 41. 60.	7477981
So is the base A B	768
To the perpendicular B C	574

By the Artificiall.

As the Radius	10.0000000
To the co-sine of B A C 41. 60.	9.8737843
So is the base A B 768,	2.8853612
To the perpendicular B C 574 :	2.7591455

Thirdly, by the natural numbers.

As the co-secant of BAC 41. 60.	15061915
Is to the co-tang. of BAC 41. 60.	11263271
So is the base AB	768
To the perpendicular BC	574

By the artificiall.

As the co-secant of BAC 41. 60.	10.1778803
Is to the co-tang. of BAC 41. 60.	10.0516645
So is the base A B 768	2.8853612
To the perpendicular B C 574	2.7591455

COROL.

COROLLARY.

Hence it is evident, that Radius is a mean proportional between the sine of an arch, and the secant complement of the same arch; also between the tangent of an arch, and the tangent of the complement of the same arch.

The second Reason.

The sines of several arches, and the secants of their complements are reciprocally proportional, that is,

As the sine of an arch or angle, is to the sine of another arch or angle: So is the secant of the complement of that other, to the co-secant of the former.

For by the foregoing Corollary, Radius is the mean proportional between the sine of any arch and the co-secant of the same arch.

Therefore, whatsoever sine is multiplied by the secant of the complement, is equall to the square of Radius; so that all rectangles made of the sines of arches and of the secants of their complements are equal one to another; but equall rectangles have their sides reciprocally proportional, by the tenth Theorem of the second Chapter. Therefore the sines of several arches, &c.

The

The third Reason.

The tangents of severall arches, and the tangents of their complements are reciprocally proportional, that is,

As the tangent of an arch or angle, is to the tangent of another arch or angle, so is the co-tangent of that other, to the co-tangent of the former.

For by the foregoing Corollary, Radius is the mean proportionall between the tangent of every arch and the tangent of his complement.

Therefore the Rectangle made of any tangent, and of the tangent of his complement, is equall to the square of Radius: so that all rectangles made of the tangents of arches, and of the tangents of their complements are equall one to another, but equal rectangles, &c. as before.

To these three reasons a fourth may be added. For in the rule of proportion; wherein there are alwayes four terms, three given, the fourth demanded; It is all one, whether of the two middle terms is put in the second or third place.

For it is all one, whether I shall say;

As 2, to 4; so 5, to 10: or say, as 2, to 5; so 4, to 10: and from hence every example in any triangle may be varied, and

thus you see the reasons of varying the termes of proportion, we come now to shew you the various proportions themselves of the severall Cases in right angled plain triangles.

Right angled plain triangles may be distinguished into seven Cases; whereof those in which a side is required, *viz.* three, may be found by a triple proportion; and those in which an angle is required, *viz.* three, may be found by a double proportion.

CASE 1.

The angles and base given, to finde the perpendicular.

First, As sine the angle at the perpendicular, is to the base: so is sine the angle at the base, to the perpendicular.

Or secondly, thus: As Radius, to the base; so tangent the angle at the base, to the perpendicular.

Or thirdly, thus: As the tangent of the angle at the perpendicular, is to the base: so is Radius, to the perpendicular.

CASE 2.

The angles and base given, to finde the hypothenusal.

First, As the sine of the angle at the perpendicular

pendicular, is to the base; so is Radius, to the hypothenusal.

Or secondly thus: As Radius, is to the base; so the secant of the angle at the base, to the hypothenusal.

Or thirdly, thus: As the tangent of the angle at the perpendicular, is to the base: so is the secant of the same angle in proportion to the hypothenusal.

CASE 3.

The angles and hypothenusal given, to finde the base.

First, As Radius, to the hypothenusal: so the sine of the angle at the perpendicular, to the base. Or secondly, thus:

As the secant of the angle at the base, to the hypothenusal: so is Radius, to the base.

Or thirdly, thus: As the secant of the angle at the perpendicular, to the hypothenusal: so the tangent of the same angle, to the base.

CASE 4.

The base and perpendicular given, to finde an angle.

First, As the base, to Radius: so the perpendicular, to the tangent of the angle at the base. Or secondly, thus:

As the perpendicular, is to Radius: so the base, to the tangent of the angle at the perpendicular.

Case

Case 5.

The base and hypothenusal given, to finde an angle.

1. As the hypothenusal, is to Radius: so is the base, to the sine of the angle at the perpendicular.

Or secondly thus, As the base is to Radius; so is the hypothenusal, to the secant of the angle at the base.

Case 6.

The base and perpendicular given, to finde the hypothenusal.

First, finde the angle at the perpendicular, by the fourth Case: Then,

As the sine of the angle at the perpendicular, is to the base: so is Radius, to the hypothenusal.

Otherwise by the Logarithmes of absolute numbers.

From the doubled Logarithme of the greater side, whether base or perpendicular, substract the Logarithme of the lesse, and to the absolute number answering to the difference of the Logarithmes adde the lesse, the half summe of the Logarithmes of the summe, and the lesse side, is the Logarithme of the hypothenusal inquired.

The

The Illustration Arithmetical.

Let the base be 768, and the perpendicular 320.

The Logarithme of 768 is	2.8853612
This Logarithme doubled is	5.7707224
From wch substr. the Log. of 320,	2.5051500
The remain. is the Log. of 18432	3.2655724

To which the lesser side being added 320, their aggregate is 2163.

The Logarithme of 2163 is	3.3350565
The Logarithme of 320 is	2.5051500
The summe is	5.8402065
The half sum is the Log. of 832,	2.9201032

which is the length of the hypothenusal inquired.

CASE 7.

The base and hypothenusal given, to finde the perpendicular.

To resolve this Probleme by the Canon, there is required a double operation: First, by the 5 Case, finde an angle. Secondly, by the first Case, finde the perpendicular.

But Mr. *Briggs* resolves this Case more readily, by the Logarithmes of the absolute numbers, *Briggs Arithmetica Logarith. cap.* 17.

Take the Logarithmes of the summe and

and difference of the hypothenusal and side given, half the summe of those two Logarithmes, is the Logarithme of the perpendicular, or side inquired.

As let the hypothen. be	832	
The side given	768	
		Logarith.
The summe is	1600	3.2041200
The difference is	64	1.8061800
The summe is,		5.0103000
The half sum is the Logarith. of 380 the side inquired.		2.5051500

The two Axiomes following are true in all plain triangles, but are chiefly intended for the oblique angled; which now we come to handle.

II. *Of plain oblique angled Triangles.*

In a plain oblique angled triangle, there are four varieties.

1. All the angles may be given, (for when two are given, the third is given by consequence) and one side, and the other two sides demanded.

2. Two sides with an angle opposite to one of them may be given, and the angle opposite to the other, with the third side are demanded. In both which cases the first Axiome is fully sufficient.

3. Two

5. Two sides with an angle comprehended by them may be given, and the other two angles with the third side demanded. For the solution whereof we will lay down this Axiome following.

The third AXIOME.

As the summe of the two sides, is to their difference: so is the tangent of half the summe of the opposite angles, to the tangent of half the difference.

Let A B C be the oblique angled triangle, in which let the side A B be continued to H, and let the line of continuation B H be made equall to B C, and B K equal to A B; then is A H the summe of the sides, A B, B C, and K H is their difference, now if you draw the lines B D and KG parallel unto A C, then shall the angle CBH be equal to the two angles of the triangle given A C B and C A B, because the angle C B A common to both is their complement to a Semicircle, and D B being parallel to C A, the angle D B H shall be equall to the angle C A B, and the angle D B C equall to the angle A C B, if therefore you let fall the perpendicular B E, and draw the periphery M E L, the right line C E shall be the tangent of half the summe of the

 angles

angles A C B and C A B, it being the tangent of half the angle C B H.

Again, if you make E F equal to D E, and draw the right line F B, then shall the angle D B F be the difference between the angles C B D and D B H, or between the angles A C B and C A B, and D E the tangent

gent of half the difference. And because the right lines A C, D B, and K G are parallel, and C D, D G, and F H are equall, and D F equal to G H, and the triangles A C H and K G H are like, and therefore; As A H is in proportion to H K: so is C H, to H G: or as A H, the summe of the sides, is in proportion to H K, their difference: so is C E the tangent of the half summe of the angles A C B and C A B, to D E, the tangent of half their difference.

Consectary.

Hence it followes, that in a plain oblique angled Triangle; if two sides and the angle comprehended by them be given; the other two angles and the third side are also given.

As in the triangle A B C, having the sides A C 189, and A B 156, whose summe is 345, and difference 33, with the angle B A C 22 degrees, 60 minutes; to finde the angle A B C or A C B. The proportion is

As the sum of the sides given 345,	2.5378190
Is to their difference 33;	1.5185139
So the tangent of half the angles at B & C 78 de. 70 m.	10.6993616
To the tangent of half their difference 25 degr. 58 minutes	9.6800565

Which being added to the half sum 78 degrees, 70 minutes, the obtuse angle at B, is 104 degrees, 28 minutes; and substracted from the half summe, it leaveth 53 degrees, 12 minutes for the quantity of the acute angle A C B.

Then to finde the third side B C, the proportion, by the first Axiome, is,

As the sine of the angle at E, is in proportion to his opposite side AB; so is the sine of the angle at A, to his opposite side B C.

4. And lastly, all the three sides may be given, and the angles may be demanded; for the solution whereof we will lay down this Axiome,

The fourth AXIOME.

As the basd, is to the summe of the sides: So is the difference of the sides, to the difference of the segments of the base.

Let B C D be the triangle, C D the base, B D the shortest side; upon the point B describe the circle A D F H, making B D the Radius thereof, let the side B C be produced to A, then is C A the summe of the sides, because B A and B D are equal, by the work, C H is the difference of the sides, C F the difference of the segments of the base.

Now

Now if you draw the right lines A F and H D, the triangles C H D and C A F shall be equiangled, because of their common angle A C F or H C D, and their equal angles C A F and H D C, which are equall, because the arch H F is the double measure to them both; and therefore, as C D, to C A; so is C H, to C F, which was to be proved.

Consectary.

Therefore the three sides of a plain oblique angled triangle being given, the reason of the angles is also given.

For first, the obliquangled triangle may be resolved into two right angled triangles, by this Axiome, and then the right angled triangles may be resolved by the first Axiome.

As in the plain oblique angled triangle, B C D, let the three sides be given, B D 189 paces, B C 156 paces, and D C 75 paces, and let the angle C B A be required.

First, by this Axiome, I resolve it into two right angled triangles; thus:

As the true base BD 189	*co. ar.*	7.7235382
Is to the sum of B C & DC 231		2.3636120
So the difference of BC&DC 81		1.9084850
To the alternate base B G 99		1.9956352

Having thus the true and the alternate base, substract the lesser 99 from the greater 189, and there rests 90, and in the middle of this remainer, that is, at 45 paces, let fall the perpendicular A C. Then in the right angled triangle A B C, we have known the base A B, *viz.* the summe of the alternate base B G 99, and the half summe of G D, that is, the length of G A 45, which added together is 144, and the

hy-

hypothenusal B C 156, hence to finde the angle at B, by the fifth Case of right angled Triangles, I say,

As the the hypothenusal B C, is to Radius; So is the base A B 144, to the sine of the angle at the perpendicular, whose complement is the angle at the base inquired.

In like manner may be found the angle at D, and then the angle B C D is found by consequence, being the complement of the other two to two right angles or 180 degrees.

CHAP. VII.

Of Sphericall Triangles.

A Sphericall Triangle is a figure described upon a Sphericall or round superficies, consisting of three arches of the greatest circles that can be described upon it, every one being lesse then a Semicircle.

2. The greatest circles of a round or Spherical superficies are those which divide the whole Sphere equally into two Hemispheres, and are every where distant from

their own centers, by a Quadrant, or fourth part of a great circle.

3. A great circle of the Sphere passing through the poles or centers of another great circle, cut one another at right angles.

4. A Spherical angle is measured by the arch of a great circle described from the angular point betwixt the sides of the triangle, those sides being continued to quadrants.

5. The sides of a Spherical triangle may be turned into angles, and the angles into sides, the complements of the greatest side or greatest angle to a Semicircle, being taken in each conversion.

It will be necessary to demonstrate this, which is of so frequent use in *Trigonometry*. In the annexed Diagram let A B C be a sphericall triangle, obtuse angled at B, let D E be the measure of the angle at A. Let F G be the measure of the acute angle at B, (which is the complement of the obtuse angle B, being the greatest angle in the given triangle) and let H I be the measure of the angle at C, K L is equal to the arch D E, because K D and L E are Quadrants, and their common complement is L D. L M is equall to the arch F G, because L G and

FM are Quadrants, and their common complement is LF. KM is equal to the arch HI, because KI and MH are Quadrants, and their common complement is KH. Therefore the sides of the triangle KLM are equal to the angles of the triangle ABC, taking for the greatest angle ABC, the complement thereof FBG.

And by the like reason it may be demonstrated, that the sides of the triangle ABC are equal to the angles of the triangle KLM. For the side AC is equall to the arch DI, being the measure of the angle DKI, which is the complement of the obtuse angle MKL. The side AB is equall to the

the arch O P, being the measure of the angle M L K. And lastly, the side B C is equall to the arch F H, being the measure of the angle L M K, for A D and C I are Quadrants: so are A P and O B, B F and C H. And C D, A O, and C F are the common complements of two of those arches. Therefore the sides of a spherical triangle may be changed into angles, and the angles into sides, which was to be demonstrated.

6. The three sides of any spherical triangle are lesse then two Semicircles.

7. The three angles of a spherical triangle are greater then two right angles, and therefore two angles being known, the third is not known by consequence, as in plain triangles.

8. If a spherical triangle have one or more right angles, it is called a right angled spherical triangle.

9. If a spherical triangle have one or more of his sides quadrants, it is called a quadrantal triangle.

10. If it have neither right angle, nor any side a quadrant, it is called an oblique spherical triangle.

11. Two oblique angles of a spherical triangle are either of them of the same kinde

kinde of which their opposite sides are.

12. If any angle of a triangle be neerer to a quadrant then his opposite side; two sides of that triangle shall be of one kinde, and the third lesse then a quadrant.

13. But if any side of a triangle be nearer to a quadrant then his opposite angle, two angles of that triangle shall be of one kinde, and the third greater then a quadrant.

14. If a sphericall triangle be both right-angled and quadrantal, the sides thereof are equall to the opposite angles.

For if it have three right angles, the three sides are quadrants; if it have two right angles, the two sides subtending them are quadrants; if it have one right angle, and one side a quadrant, it hath two right angles and two quadrantal sides, as is evident by the third Proposition. But if two sides be quadrants, the third measureth their contained angle, by the fourth proposition. Therefore for the solution of these kindes of triangles, there needs no further rule; But for the solution of right angled, quadrantall, and oblique sphericall triangles there are other affections proper to them, which are necessary to be known as well as these general affections

common

common to all spherical triangles. The affections proper to right angled and quadrantal triangles we will speak of first.

CHAP. VIII.

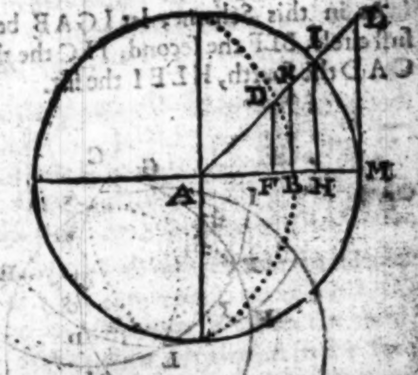
Of the affections of right angled Sphericall Triangles.

IN all spherical rectangled Triangles, having the same acute angle at the base: The sines of the hypothenusals are proportional to the sines of their perpendiculars. As in the annexed diagram, let A D B represent a spherical triangle, right angled at B; so that A D is the sine of the hypothenusal, A B the sine of the base, and D B is the perpendicular. Then is D A B the angle at the base, and I H the sine, and L M the tangent thereof: Also D F is the sine of the perpendicular, D B and K B is the tangent thereof: I say then, As A D, is to F D: So is A I, to I H, by the *tenth* Theoreme of the second Chapter.

And because it is all one, whether of the mean proportionals be put in the second

place;

place; therefore I may say: As AD, the sine of the hypothenusal, is in proportion to AI Radius: So is FD, the sine of the perpendicular, to IH the sine of the angle at the base.



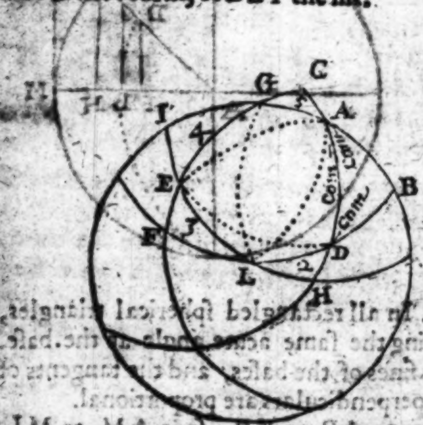
2. In all rectangled spherical triangles, having the same acute angle at the base. The sines of the bases, and the tangents of the perpendiculars are proportional.

For as AB, to KB; so is AM, to ML, by the 26th Theorem of the second Chapter: or which is all one; As AB, the sine of the base, is in proportion to AM Radius: so is BK, the tangent of the perpendicular, to ML, the tangent of the angle at the base.

3. If

3. If 5 circles of the Sphere be so ordered, that the first intersect the second, the second the third, the third the fourth, the fourth the fift, and the fift the first at right angles; the right angled triangles made by their intersections do all consist of the same circular parts.

As in this Scheme, let I G A B be the first circle, B L F the second, F E C the third, C A D the fourth, H L E I the fift.



Then do these five circles retain the conditions required. The first intersecting the second in B, the second the third in F, the third the fourth in C, the fourth the

fift

fift in H, the fift the firſt in I. And theſe interſections at B, F, C, H, I, are at right angles; therefore I ſay, the right angled triangles made by the interſections of theſe circles; namely, A B D, D H L, L F E, E G I, and G C A do all conſiſt of the ſame circular parts; for the circular parts in every of theſe triangles are, as here appeareth.

In	ABD	are	AB	BD	cBDA	cAD	cDAB
	DHL		cHLD	cLD	cLDH	DH	HL
	LFE		cōELF	LF	FE	cōFEL	cEL
	EGI		IG	cōIGE	cGE	cōGEI	IE
	GCA		cGA	cAGC	GC	CA	cCAG

Where you may obſerve, that the ſide AB in the firſt triangle is equal to *compl.* H L D in the ſecond, or *compl.* E L F in the third, or I G in the fourth, or *com.* G A in the fift; and ſo of the reſt.

To expreſſe this more plainly, A B in the firſt triangle is the complement of the angle H L D in the ſecond, or the complement of the angle E L F in the third, or the ſide I G in the fourth, or the complement of the hypothenuſal G A in the fift. And from theſe premiſes is deduced this univerſall propoſition.

4. The

4. The sine of the middle part and Radius are reciprocally proportional, with the tangents of the extreams conjunct, and with the co-sines of the extreams disjunct.

Namely; As the Radius, to the tangent of one of the extreames conjoyned: so is tangent of the other extream conjoyned, to the sine of the middle part.

And also; As the Radius, to the co-sine of one of the extreams dis-joyned: so the co-sine of the other extream dis-joyned, to the sine of the middle part.

Therefore if the middle part be sought, the Radius must be in the first place, if either of the extreams; the other extream must be in the first place.

For the better Demonstration hereof, it is first to be understood, that a right angled Spherical Triangle hath five parts besides the right angle. As the triangle ABD in the former Diagram, right angled at B, hath first, the side AB: secondly, the angle at A: thirdly, the hypothenusal AD: fourthly, the angle ADB: fifthly, the side DB. Three of these parts which are far-

thest

thest from the right angle, we mark or note by their complements to a quadrant. As the angle B A D we account as the complement to the same angle. For A D we write comp. A D, and for A D B we write compl. A D B. But the two sides D B and A B being next to the right angle, and so write complement B A D are not noted by their complements. Of these five parts, two are alwayes given to finde a third, and of these three one is in the middle, and the other two are extreams either adjacent to that middle one, or opposite to it. If the parts given and required are all conjoyned together, the middle is the middle part conjunct, and the extreams the extream parts conjunct. If again any of the parts given or required be dis-joyned, that which stands by it self is the middle part dis-joyned, and the extreames and extream parts dis-joyned. Thus, if there were given in the triangle A B D, the side A B, the angle at A, to finde the hypothenusal A D, there the angle at A is in the middle, and the sides A D and A B are adjacent to it, and therefore the middle part is called the middle conjunct, and the extreames are the extreames conjunct; but if there were given the side A B, the hypothenusal A D, to finde the

the angle at D, here A B is the middle part dis-junct, because it is dis-joyned from the side A D by the angle at A, and from the angle at D by the side D B, for the right angle is not reckoned among the circular parts, and here the extreams are extreams dis-junct.

These things premised, we come now to demonstrate the proposition it self, consisting of two parts: first, we will prove, that the sine of the middle part and Radius are proportional with the tangents of the extreams conjunct.

The middle part is either one of the sides, or one of the oblique angles, or the hypothenusal.

CASE I.

Let the middle part be a side, as in the right angled spherical triangle A B D of the last diagram, let the perpendicular A B be the middle part, the base D B and *comp.* A the extreams conjunct, then I say, that the rectangle of the sine of A B and Radius is equal to the rectangle of the tangent of D B, and the tangent of the complement of D A B: for, by the second proposition of this Chapter, As the sine of A B, is in proportion to Radius: so is the tangent of

D B,

DB, to the tangent of the angle at A: Therefore if you put the third term in the second place, it will be, as the sine of AB, to the tangent of DB: so is the Radius, to the tangent of the angle at A. But Radius is a mean proportional between the tangent of an arch, and the tangent of the complement of the same arch, by the Corollary of the first reason of the second Axiome of plain Triangles; and therefore as Radius, is to the tangent of the angle at A; so is the tangent complement of the same angle at A unto Radius: Therefore as the sine of AB is in proportion to the tangent of DB; so is the co-tangent of the angle at A, to Radius: and therefore the rectangle of AB & Radius, is equall to the rectangle of the tangent of DB, and the co-tangent of the angle at A.

CASE 3.

Let the middle part be an angle, as in the triangle DHL of the former Diagram, and let compl. HLD be the middle part, HL and compl. LD the extremes conjunct; then I say, that the rectangle made of the co-sine of HLD and Radius, is equal to the rectangle of the tangent of HL and the co-tangent of LD. For, by the third pro-

proposition of this Chapter, compl. H L D is equal to A B, and compl. L D to D B; and H L to compl. D A B; and here we have proved before, that the rectangle of the sine of A B and Radius, is equal to the rectangle of the tangent of D B, and the co-tangent of the angle at A; therefore also the rectangle of the co-sine of H L D and Radius, is equal to the rectangle of the co-tangent of L D, and the co-tangent of H L.

Case 3.

Let the middle part be the hypothenusal, as in the triangle G C A, let compl. A G be the middle part, compl. A G C, and compl. C A G the extreams conjunct; then I say, that the rectangle of the co-sine of A G and Radius, is equal to the rectangle of the co-tangent of A G C, and the co-tangent of C A G: for we have proved before, that the rectangle of the sine of A B and Radius is equal to the rectangle of the tangent of D B and the co-tangent of D A B, but, by the third proposition of this Chapter, compl. A G is equal to A B, compl. A G C to D B, and compl. C A G to compl. D A B; therefore also the rectangle of the co-sine of A G and Radius, is equal to the rectangle

rectangle of the co-tangent of AGC and the co-tangent of CAG, which was to be proved.

It is further to be proved, that the sine of the middle part and Radius are proportional with the co-sines of the extreams dis-junct. Here also the middle part is either one of the sides, or the hypothenusal, or one of the oblique angles.

CASE I.

Let the middle part be a side: as in the triangle ABD, let DB be the middle part, compl. AD and compl. A the opposite extreams: then I say, that the rectangle of the sine of BD and Radius is equal to the rectangle of the sine of AD, and the sine of the angle at *A*; for, by the first proposition of this Chapter, as the sine of *A*D, is to Radius; so is the sine of DB, to the sine of the angle at *A*. Therefore, the rectangle of the sine of DB and Radius, is equal to the rectangle of the sine of *A*D and the sine of the angle at *A*.

CASE 2.

Let the hypothenusal be the middle part; as in the triangle DHL, let compl. LD be

be the middle part, D H and H L the extreams dis-junct. Then I say, that the rectangle of the co-sine of L D and Radius is equal, to the rectangle of the co-sine of D H and the co-sine of H L: for compl. L D is equal to D B, and D H is equall to compl. A D, and H L to compl. D A B, by the third proposition of this Chapter; therefore the rectangle of the co-sine of L D and Radius, is equal to the rectangle of the co-sine of D H and the co-sine of H L.

Case 3.

Let one of the oblique angles be the middle part, as in the triangle I E G, let compl. I G E be the middle part: then I say, that the rectangle of the co-sine of I G E and Radius is equal to the rectangle of the sine of G E I and the co-sine of I E: for compl. I G E is equall to D B, and G E I is equal to A D, and E I to compl. D A B.

5. In any Spherical triangle, the sines of the sides are proportional to the sines of their opposite angles.

Let A B C be a spherical triangle, right angled at C, then let the sides A B, A C, and C B be continued to make the quadrants A E, A F, and C D, and from the pole

pole of the quadrant A F, to wit, from the point D, let be drawn down the other quadrants D F and D H; so there is made three new triangles B D E, G D E, and the obliquangled triangle B D G. I say, in the right angled triangle A B C, that the sine of the side A B is in proportion to the sine of his opposite angle A C B: as the sine of the side A C, is to his opposite angle A B C; or as B C, to B A C: likewise in the obliquangled spherical triangle B D G, I say, that as B G, is to B D G: so is B D, to B G D; or so is D G, to D B G.

For first, in the right angled triangle A B C, the angle A C B and the arch A E are of the same quantity, to wit, quadrants, so likewise the angle B A C and the arch

E F

E F are of the same quantity, it being the measure of the said angle. Now then as *A* B, to *A* E; so is B C, to E F, by the first proposition of this Chapter: therefore also as *A* B, to *A* C B; so is B C, to B *A* C. Then in the obliquangled Triangle B D G, because, by the demonstration of right angled triangles, they are as D B, to D E B; so is D E, to D B E: and as D G, to D E G: so is D E, to D G E, or to D G B. Therefore changing of the proportionall termes, it shall be, as D G, to D B: so is D B E, or D B G, to D G B, which was to be demonstrated.

These foundations being thus laid, the businesse of right angled sphericall triangles is easily dispatcht. And the proportions to be used in every case may be discovered either by the first, second and fift propositions; or by the fourth proposition only. The severall cases in a right angled sphericall triangle are sixteen in number, whereof six may be resolved by the first proposition: seven by the second, and three by the fift; an example in each will suffice.

In the triangle A B C, let there be given the hypothenusal A B, and the perpendicular B C, to finde the base A C; then by the first proposition, the Analogie is,

 As

As the co-ſine of the perpendicular, is to Radius: ſo is the co-ſine of the hypothenuſal, to the co-ſine of the baſe.

2. Let there be given the baſe *A* C, and the angle at the baſe B *A* C, to finde the perpendicular B C, by the ſecond propoſition, the analogie is:

As Radius, to the ſine of the baſe; ſo is the tangent of the angle at the baſe, to the tangent of the perpendicular.

3. Let there be given the hypothenuſal *A* B, the angle at the baſe B *A* C, to finde the perpendicular B C, by the fifth propoſition, the analogie is:

As Radius, to the ſine of the hypothenuſal: ſo is the ſine of the angle at the baſe, to the ſine of the perpendicular; and ſo of the reſt.

By the fourth or univerſall Propoſition, the proportions for right angled ſphericall triangles may be found two wayes:

Firſt, by the equality of the Sines and Tangents of the circular parts of a triangle, that is, of the Logarithmes of the natural, thus by the univerſal propoſition in the aforeſaid triangle *A* B C, the hypothenuſal *A* B, and the angles at *A* and B being noted by their complements, I ſay.

1. The ſine of *A* C added to Radius; is

equal to the ſine of *A* B added to the ſine of the angle at C.

2. The coſine of *A* added to Radius is equal to the co-ſine of B C added to the ſine of the angle at B.

3. The co-ſine of *A* B added to Radius, is equall to the co-ſine of *A* C added to to the co-ſine of B C.

4. The co-ſine of *A* B added to Radius is equal to the co-tangent of *A*, added to the co-tangent of the angle at B.

5. The coſine of the angle at B added to Radius is equal to the tangent of B C, added to the cotangent of *A* B.

6. The ſine of B C added to Radius is equal to the co-tangent of the angle at B added to the tangent of *A* C.

And thus he that liſteth may ſet down the equality of the ſines and tangents of the other ſides and angles, and ſo there will be ten in all; but theſe may here ſuffice: for to theſe may the ſixteen caſes of a right angled ſpherical triangle be reduced; namely, three to the firſt, three to the ſecond, two to the third, two to the fourth, three to the fiſt, and three to the ſixt.

As admit there were given the hypothenuſal B *A*, and the angle at B, to finde the baſe *A* C; then, by the firſt, ſeeing that the

the ſine of *A* B added to the ſine of the angle at B, is equal to the ſine of *A* C added to Radius. Therefore, if working by natural numbers I multiply the ſine of *A* B by the ſine of B, and divide the product by Radius, the remainer will be the ſine of *A* C.: and working by Logarithmes, if from the ſumme of the ſines of *A* B and B I ſubſtract Radius, the reſt is the ſine of *A* C.

Secondly, admit there were given A B and A C, to finde B, then ſeeing that the ſine of A C added to Radius is equal to the ſines of A B and B. Therefore, if working by naturall numbers I multiply the ſine of *A* C by Radius, and divide the product by *A* B, the remainer is the ſine of B. Or working by Logarithmes, if from the ſum of the ſines of *A* C and Radius, I ſubſtract the ſine of *A* B, the remainer will be the ſine of B.

Or thirdly, if there were given A C and the angle at B, to finde A B: then foraſmuch as A C and Radius is equal to the ſines of A B and B, therefore if working by natural numbers I multiply A C by the Radius, and divide the product by the ſine of B, the remainer is the ſine of A B: Or working by Logarithmes, if from the ſine

of A C and Radius, I substract the sine of B the remainer is the sine of A B: and so of the rest.

Which that you may the better perceive, I have here added in expresse words, the Canons or rules of the proportions of the things given and required in every of the sixteen cases of a right angled sphericall triangle, as they are collected from the Catholick Proposition. And here the side subtending the right angle we call the hypothenusal, the other two containing the right angle we may call the sides; but for further distinction, we call one of these containing sides (it matters not which) the base, and the other the perpendicular.

The base and angle at the base given, to finde

1. *The perpendicular.*] As Radius, to the sine of the base; so is the tangent of the angle at the base, to the tangent of the perpendicular.

2. *Angle at the perpendicular.*] As Radius, to the co-sine of the base; so the sine of the angle at the base, to the co-sine of the angle at the perpendicular.

3. *Hypothenusal.*] As Radius, to the co-sine of the angle at the base: so the co-tangent

tangent of the base, to the co-tangent of the hypothenusal.

The perpendicular and angle at the base given, to finde

4. *Angle at perpend.*] As the co-sine of the perpendicular, to Radius; so the co-sine of the angle at the base, to the sine of the angle at the perpendicular.

5. *Hypothenusal.*] As the sine of the angle at the base, to Radius; so the sine of the perpendicular, to the sine of the hypothenusal.

6. *The Base.*] As Radius, to the co-tangent of the angle at the base; so is the tangent of the perpendicular, to the sine of the base.

The hypothenusal and angle at the base given, to finde

7. *The base.*] As Radius, to the co-sine of the angle at the base; so the tangent of the hypothenusal, to the tangent of the base.

8. *Perpendicular.*] As Radius, to the sine of the hypothenusal, so the sine of the angle at the base, to the sine of the perpendicular.

9. *Angle at perpend.*] As Radius, to the co-

co-sine of the hypothenasal; so the tangent of the angle at the base, to the co-tangent of the angle at the perpendicular.

The base and perpendicular given, to finde

10. *Hypothenusal.*] As Radius, to the co-sine of the perpendicular: so the co-sine of the base, to the co-sine of the hypothenusal.

11. *Angle at the base*] As Radius, to the sine of the base: so is the co-tangent of the perpendicular, to the co-tangent of the angle at the base.

The base and hypothenusal given, to finde the

12. *Perpendicular.*] As the co-sine of the base, to Radius; so the co-sine of the hypothenusal, to the co-sine of the perpendicular.

13. *Angle at the base.*] As Radius, to the tangent of the base; so the co-tangent of the hypothenusal, to the co-sine of the angle at the base.

14. *Angle at the perpend.*] As the sine of the hypothenusal, to Radius; so the sine of the base, to the sine of the angle at the perpendicular.

The

The angles at the base and perpendicular given, to finde

15. *The perpendicular.*] As the sine of the angle at the perpendicular, is to Radius: so the co-sine of the angle at the base, to the co-sine of the perpendicular.

16. *The hypothenusal.*] As Radius, to co-tangent of the angle at the perpendicular; so the co-tangent of the angle at the base, to the co-sine of the hypothenusal.

Secondly, the proportions of all the cases of a right angled spherical triangle, may by the aforesaid Catholick Proposition be known thus: If the middle part be sought, put the Radius in the first place; If either of the extreams, the other extream put in the first place.

And note, that when a complement in the proposition doth chance to concur with a complement in the circular parts, you must take the sine it self, or the tangent it self, because the co-sine of the co-sine is the sine, and the co-tangent of the co-tangent is the tangent.

As in the following triangle A B C, let there be given the base A B, and the angle at C, to finde the hypothenusal B C. Here

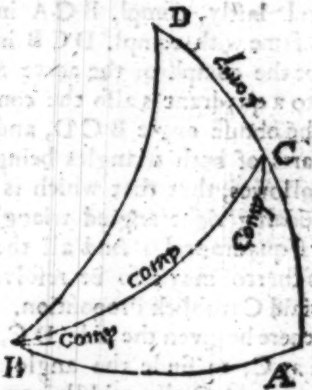
A B is the middle part, B C and C are the opposite extreams, or the extreams disjunct. Now because the extream B O is sought, therefore I must put the other extream, that is, the angle at C, in the first place; and because that angle, as also the side sought are noted by their complements, therefore I must not say: As the co-sine of the angle at C, is to Radius: so is the sine of the base A B, to the co-sine of the hypothenusal B C: but thus;

As the sine of the angle at the perpendicular A C B, is to Radius; so is the sine of the base A B, to the sine of the hypothenusal B C. The like is to be understood of the rest.

Thus much concerning right angled spherical triangles: as for Quadrantal there needs not much be said, because the circular parts of a quadrantal triangle, are the same with the circular parts of a right angled triangle adjoyning.

As let A B C be a triangle, right angled at A, and let one of the sides thereof; namely, A C be extended, till it become a quadrant, that is to D; then draw an arch from D to B; then is D B C a quadrantal triangle, to which there is a right angled triangle adjoyning, as A B C. I say therefore

fore that the circular parts of the quadrantal triangle B C D are the same with the circular parts of the right angled triangle A B C ; for the circular parts of either of them are as here appeareth.



The five circular parts of the triangle.

ABC are AC ABcōABCcōBGcō.BCA
BCDarecomCD CDB DBCcōBCcōBCD

Where it is evident, that A D and D B being quadrants, D B A is a right angle, and B A is the measure of the angle at D,

so that the side A C in the one is equall to compl. C D in the other: and the side AB in the one is equal to the angle B D C in the other: and compl. A B C in the one is equal to D B C in the other, and compl. B C in the one is the same with B C in the other: and lastly, compl. B C A in the one is the same with compl. D C B in the other; for the compl. of the acute angle A B C unto a quadrant is also the complement of the obtuse angle B C D, and the circular parts of both triangles being the same, it followes, that that which is here proved touching right angled triangles is also true of quadrantal. And all the sixteen cases thereof may also be resolved by the aforesaid Catholick Proposition.

As let there be given the side D C, and the angle at C, to finde the angle at D, then is the side D C the middle, and the angles at D and C are extreams adjacent; now because the angle at D, one of the extreams is sought, we must put the other extream, to wit, the angle at C in the first place, and that is noted by its complement: and therefore the Analogie is;

As the co-tangent of the angle at C, to Radius; so the co-sine of D C, to the tangent of the angle at D: and so of the rest; and

and what is said of the addition of the artificial numbers is to be understood of the rectangles of the natural.

CHAP. IX.

Of Oblique angled Sphericall Triangles.

IN an obliquangled spherical triangle, there are twelve Cases; two whereof, that is, those wherein the things given and required are opposite, may be resolved by the fist proposition of the last Chapter.

Case I.

Two angles with a side opposite to one of them being given, to finde the side opposite to the other.

As in the triangle A B C, let there be given the side B C, with his opposite angle at A, and the angle A B C, to finde the side A C. I say then, by the fist proposition of the last Chapter:

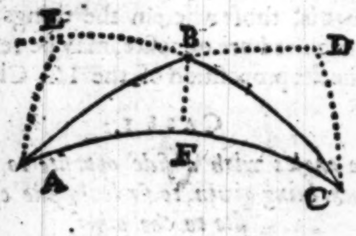
As the sine of the angle at A, is to the sine

fine of his opposite side B C : so is the fine of the angle at B, to the fine of his opposite side A C.

Case 2.

Two sides with an angle opposite to one of them being given, to finde an angle opposite to the other.

As in the triangle A B C, let there be given the sides B C and A C, with the angle at A, to finde the angle at B : I say then, by the fist proposition of the last Chapter :



As the fine of B C, to the fine of his opposite angle at A : so is the fine of A C, to the fine of his opposite angle B.

Other eight cases must be resolved by the aid of two Analogies at the least, and that by reducing the triangle proposed to two right angled triangles, by a perpendicular let

let fall from one of the angles to his opposite side, which perpendicular falls sometimes within, sometimes without the triangle.

If the perpendicular be let fall from an obtuse angle, it falleth within, but if it fall from an acute angle, it falls without the triangle: however it falleth, it must be alwayes opposite to a known angle.

For your better direction, in letting fall the perpendicular take this generall rule.

From the end of a side given, being adjacent to an angle given, let fall the perpendicular.

As in the triangle, A B C, if there were given the side A B, and the angle at A: by this rule the perpendicular must fall from B upon the side A C; but if there were given the side A C, and the angle at A; then A B must be produced to D, and the perpendicular must fall from C upon the side A D. Thus shall we have two right angled triangles, and the side or angle required may easily be resolved by the Catholick Proposition.

As suppose there were given the side A B, the angles at A and C, and required the side A C; then the perpendicular must fall from B upon the side A C, as in the first

tri-

triangle, and divide the oblique triangle A B C into two right angled triangles, to wit, A B F and B F C. And in the triangle A B F we have given the side A B, and the angle at A, to finde the base A F, for which the analogie, by the Catholick Proposition, is,

As the co-tangent of A B, to Radius : so is the co-sine of the angle at A, to the tangent of A F : that is, by the seventh case of right angled triangles.

Secondly, by the eighth case, finde the perpendicular B F. Lastly, in the triangle B F C, having the perpendicular B F, and the angle at C, by the sixt case of right angled spherical triangles, you may finde the base F C, which being added to A F, is the side A C.

But thus there are three operations required; whereas it may be done at two: for the obliquangled triangle being reduced into two right angled triangles, by letting fall a perpendicular, as before : the hypothenusal in one of the right angled triangles will be correspondent to the hypothenusal in the other, and the base in the one to the base in the other; and so the other parts.

Then in one of these right angled triangles

gles (which for distinction sake we call the first) there is given the hypothenusal and angle at the base, whereby may be found the base or angle at the perpendicular, as occasion requires; by the seventh or ninth cases of right angled triangles. And this is the first operation.

For the second, there must (of the things thus given and required) two things in one triangle, be compared to two correspondent things in the other triangle, which two in each with the perpendicular make three things in each triangle, either adjacent, that is, lying together, or opposite of which three the perpendicular is alwayes one of the extreams, and the thing required one of the other extreams.

Thus in the triangle A B F, if there were given A F and B F, to finde A B: A B is the middle part, A F and B F are opposite extreams; and therefore by the Catholick Proposition:

Radius added to the co-sine of A B, is equal to the co-sines of A F and B F.

Then in the triangle B F C, if there were given B F and F C, to finde B C: BC will be the middle part, B F and F C opposite extreames; and therefore by the Catholik Proposition,

The

The co-sines of B F and F C are equall to the co-sine of B C and Radius.

But if from equal things we take away equal things, the things remaining must needs be equal; if therefore we take away the Radius, and co-sine B F in both these proportions, it followes, that the co-sine of A B added to the co-sine of F C is equal to the cosine of B C added to the co-sine A F. And therefore, the middle part A B in the first, and the extream F C in the second, is equall to the middle part B C in the second, and the extream A F in the first: or thus;

As the middle part in the first triangle, is in proporion to the middle part in the second: so is the extream in the first, to the extream in the second.

Thus by the Catholick Proposition, and the help of this, the eight cases following may be resolved. In the exemplification whereof this sign + signifies addition.

By

By the Catholick Propoſition, it is evident that

$$1\left\{\begin{array}{l}\text{Rad.} + cs\text{AB}\\ cs\text{BF} + cs\text{FC}\end{array}\right\}\begin{array}{l}\text{is e-}\\ \text{qual to}\end{array}\left\{\begin{array}{l}cs\text{AF} + cs\text{FB}\\ cs\text{BC} + \text{Rad.}\end{array}\right.$$

$$2\left\{\begin{array}{l}\text{Rad.} + s\text{AF}\\ t\text{FB} + ct\text{C}\end{array}\right\}\begin{array}{l}\text{is e-}\\ \text{qual to}\end{array}\left\{\begin{array}{l}ct\text{A} + t\text{FB}\\ s\text{FC} + \text{Rad.}\end{array}\right.$$

$$3\left\{\begin{array}{l}\text{Rad.} + cs\text{A}\\ cs\text{FB} + s\text{FBC}\end{array}\right\}\begin{array}{l}\text{is e-}\\ \text{qual to}\end{array}\left\{\begin{array}{l}s\text{ABF} + cs\text{FB}\\ cs\text{C} + \text{Rad.}\end{array}\right.$$

$$4\left\{\begin{array}{l}\text{Ra.} + cs\text{ABF}\\ t\text{FB} + ct\text{BC}\end{array}\right\}\begin{array}{l}\text{is e-}\\ \text{qual to}\end{array}\left\{\begin{array}{l}ct\text{AB} + t\text{FB}\\ cs\text{FBC} + \text{Ra.}\end{array}\right.$$

Then taking from either ſide tangent F B and Radius; or co-ſine F B and Radius, it followes, by the former propoſition, that

1. *cs*AB + *cs* FC is equall to *cs*BC + *cs*AF.
2. *s* AF + *ct* C is equall to *s* FC + *ct* A.
3. *cs* A + *s*FBC is equall to *cs* C + *s*ABF.
4. *cs*ABF + *ct* BC is equal to *cs*FBC + *ct*AB.

For ſeeing that A F and F B are oppoſite extreams to A B, as C F and F B are to B C: therefore,

1. As *cs* A F, to *cs* F C; ſo is *cs* A B, to *cs* B C: that is, As co-ſine the firſt baſe, to co-ſine the ſecond; ſo co-ſine the firſt hypothenuſal, to co-ſine the ſecond. And this

ſerves

serves for the third and seventh cases following.

And seeing that A and FB are adjacent extreams to AF: as C and FB are to FC: therefore,

2. As s AF, to s FC; so ct A, to ct C: that is, as the sine of the first base, to the sine of the second; so co-tangent the first angle at the base, to co-tangent the second, which serves for the fourth and tenth cases.

Again, seeing that ABF and FB are opposite extreams to A, as CBF and FB are to C: therefore,

3. As s ABF, to s CBF; so cs A, to cs C: that is, as the sine of the first angle at the perpendicular, to the sine of the second; so co-sine the first angle at the base, to co-sine the second: which serves for the fifth and ninth cases.

Lastly, seeing AB and FB are adjacent extreams to ABF, as BC and FB are to CBF: therefore,

4. As cs ABF, to cs CBF; so ct AB, to ct BC: that is, as co-sine the first angle at the perpendicular, to co-sine the second; so co-tangent the first hypothenusal, to co-tangent the second: this serves for the sixth and eighth cases following. And this foundation being thus laid, we come now to the

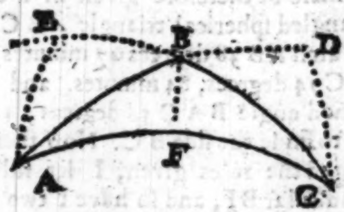
the ſeverall Caſes thereon depending.

Case 3.

Two ſides and their contained angle given, to finde the third ſide.

First, by the ſeventh caſe of right angled triangles, the analogie is:

As Radius, to the co-ſine of the angle at the baſe: ſo is the tangent of the hypothenuſal, to the tangent of the baſe, or firſt arch. Which being added to or ſubſtracted from the baſe given, according to the following direction, giveth the ſecond arch.



If the perpendicular fall } Within the triangle, ſubtract *A* F the baſe found from *A* C the baſe given, the remainer is E C, the ſecond arch.

If

If the perpendicular fall { Without, and the contained angle obtuse, adde the arch found to the arch given, and their aggregate is the second arch.
Without, and the contained angle acute, substract the arch given from the arch found, the remainer is the second arch.

Then, by the first Consectary aforegoing say: as the co-sine of the first base, to the co-sine of the second; so the co-sine of the first hypothenusal, to the co-sine of the second: but this we will illustrate by example.

Let there be therefore given in the oblique angled spherical triangle ABC, the side or arch AB 38 degrees 47 minutes, the side AC 74 degrees, 84 minutes, and their contained angle BAC 56 degrees, 44 minutes, to finde the side BC. Now then according to the rules given, I let fall the perpendicular BF, and so have I two right angled triangles, the triangle ABF and the triangle BFC. In the triangle ABF, we have the hypothenusal AB 38 degrees, 47 minutes, and the angle at the base BAF 56 degrees, 44 minutes, to finde the base AF. First therefore I say,

As

As the Radius 90, 10,000000
Is to the co-sine of BAC 56. 44. 9.742576
So is the tangent of A B 38. 47. 9.900138
To the tangent of A F 23. 72. 9.642714

Now because the perpendicular falls within the triangle, I substract A F 23 degrees, 72 minutes from A C 74 degrees, 84 min. and there remains F C 51 degrees, 12 minutes, the second arch. Hence to finde B C, I say;

As the co-sine of A F 23. 72. *co.ar.* 0.038331
Is to the co-sine of F C 51. 12. 9.797746
So is the co-sine of A B 38. 47. 9.893725
To the co-sine of B C 57. 53. 9.729802

2 Example.

In the same triangle, let there be given the side A B 38 degr. 47 min. the side B C 57 degr. 53 min. and their contained angle A B C 107 deg. 60 min. and let the side A C be sought. First, let fall the perpendicular D C, and continue the side A B to D, then in the right angled triangle B D C, there is given the angle D B C 72 deg. 40 min. the complement of the obtuse angle A B C, and the hypothenusal B C 57 degrees 53 minutes: to finde B D, I say first;

As

As the Radius 90,	10.000000
Is to the co-sine of DBC 72.40.	9.480539
So is the tangent of BC 57.53.	10.196314
To the tangent of BD 25. 42.	9.676853

Now because the perpendicular falls without the triangle, and the contained angle obtuse, I adde BD 25 degrees, 42 minutes to *A*B 38 deg. 47 min. and their aggregate is *A*D 63 deg. 89 min. the second arch: hence to finde AC, I say,

As the co-sine of BD, 25. 42.	0.044223
Is to the co-sine of 63. 89.	9.643547
So is the co-sine of BC 57. 53.	9.729859
To the co-sine of AC 74. 84.	9.417629

3 *Example.*

In this triangle, let there be given the side BC 57 deg. 53 min. the side AC 74 deg. 84 min. and their contained angle ACB 37 deg. 92 min. and let the side AB be sought: First, I let fall the perpendicular AE, and the side BC I continue to E, then in the right angled triangle AEC, we have known the angle ACE, and the hypothenusal AC, to finde EC, I say then:

As

As the Radius 90,	10,000000
Is to the co-sine of ACE 37. 92.	9.897005
So is the tangeent of AC 74. 84.	10.567120
To the tangent of EC 71. 5.	10.464125

Now because the perpendicular falls without the triangle, and the contained angle acute, I substract the arch given BC 57 degrees 53 minutes from EC 71 degrees 5 minutes, the arch found, and their difference 13 deg. 52 min. is EB, the second arch. Hence to finde AB, I say:

As the co-sine of EC 71.5. *co.ar.*	0.488462
Is to the co-sine of EB 13.52.	9.987795
So is the co-sine of AC 74.84.	9.417497
To the co-sine of AB 38. 47.	9.893753

CASE 4.

Two sides and their contained angle given, to finde one of the other angles.

First, by the seventh case of right angled spherical triangles, I say: As Radius, to the co-sine of the angle at the base; so is the tangent of the hypothenusal, to the tangent of the base, or first arch: which being added to, or substracted from the base given, according to those directions given in the third case, giveth the second arch; then

then by the second Consectary of this Chapter, the proportion is:

As the sine of the first base, to the sine of the second: so is the co-tangent of the first angle at the base, to the co-tangent of the second.

1 Example.

Thus if there were given, as in the first example of the last case, the side *A*B 38 degrees, 47 minutes, the side *A*C 74 degrees, 84 minutes, and their conteined angle B *A* C 56 degrees, 44 min. and *A* C B, the angle sought, the first operation will in all things be the same, and *A* F 23 degrees, 71 minutes, the first arch, F C 51 degrees, 12 minutes, the second; hence to finde the angle A C B, I say:

As the sine of AF 23. 72. *co.ar.*	0.395486
To the sine of F C 51. 12.	9.891237
So is the co-tang. of BAC 56. 44.	9.821771
To the co-tangent of ACB 37.92.	10.108494

There being no other variation in this case then what hath been shewed in the former, one example will be sufficient.

Case

Case 5.

Two angles, and the side between them given, to finde the third angle.

First, by the ninth case of right angled spherical triangles, the proportion is; As Radius, to the co-sine of the hypothenusal; so the tangent of the angle at the base, to the co-tangent of the angle at the perpendicular, which being added to, or substracted from the other given angle, according to the following direction, giveth the second arch.

If the perpendicular fall

- Within the triangle, substract the angle found from the angle given, the remainer is the second arch.
- Without, and both the angles given acute, substract the angle given from the angle found, and the remainer is the second arch.
- Without, and one of the angles given be obtuse, adde the angle found to the angle given, & their aggregate is the second arch.

Then, by the third Consectary of this Chapter, the analogie is; As the sine of the first angle at the perpendicular, to the

fine of the second angle found: so is the co-fine of the first angle at the base, to the co-fine of the second.

1 *Example.*

In the triangle *A* B C, let there be given the angles B *A* C 56 degrees 44 minutes, and *A* B C 107 degrees, 60 minutes, and the side between them *A* B 38 degrees 47 minutes, to finde the angle *A* C B. First, let fall the perpendicular B F, and then in the right angled spherical triangle *A*BF we have known the angle at the base B*A*F, and the hypothenusal *A* B, to finde the angle at the perpendicular *A* B F. First, then I say;

As the Radius 90,	10.000000
To the co-fine of *A* B 38. 47.	9.893725
So is the tangent of B*A*F 56.44.	10.178229
To the co-tangent of *A*BF 40.28.	10.071954

Now because the perpendicular falls within the triangle, therefore I substract the angle found *A* B F 40 degrees 28 minutes, from *A* B C 107 degrees 60 minutes, the angle given, and their difference 67 degr. 32 min. is the angle F B C, the second arch: hence to finde the angle A C B, I say;

As

As the sine of A B F 40. 28. *co,ar.* 0.189415
To the sine of F B C 67. 32. 9.965047
So is the co-sine of B A F 56. 44. 9.742576
To the co-sine of A C B 37. 92. 9.897038

Let there be given, as before, the two angles B A C and A B C, with the side between them A B, to finde the angle A C B, and let the perpendicular E A, and let the side B C be continued to E, then in the right angled triangle A E B we have known the hypothenusal A B 38 degrees, 47 minutes, and the angle at the base A B E 72 degrees, 40 minutes, the complement of the obtuse angle A B C, to finde the angle E A B. First then I say:

As the Radius 90. 10,000000
To the co-sine of A B 38. 47. 9.893725
So is the tangent of ABE 72. 40. 10.498641
To the co-tangent of EAB 22.6. 10,392366

And because the perpendicular falls without the triangle, and one of the angles given obtuse, I adde the angle found E A B 22 degrees 6 minutes to the angle given B A C 56 degrees, 44 minutes, and their aggregate 78 degrees 50 minutes is the angle E A C, the second arch; and hence to finde the angle at C, I say, as before,

As the sine of E A B 22.6. *co. ar.* 0.415307
To the sine of E A C 78.59. 9.991197
So is the co-sine of A B E 72.40. 9.480538
To the co-sine of A C B 37.92. 9.897032

3. Example.

Let there be given the angles B A C 56 degrees 44 minutes, and A C B 37 degrees, 92 minutes, with their contained side A C 74 degres, 84 minutes, to finde the angle A B C, let fall the perpendicular C D, and let the side A B be continued to D, then in the right angled triangle A D C, we have known the hypothenusal A C, and the angle at the base D A C, to finde A C D; first, then I say;

As the Radius 90 . . . 10.000000
To the co-sine of AC 74. 84. . 9.417497
So is the tangent of DAC 56.44. 10.178229
To the co tangent of ACD 68.48. 9.595726

Now because the perpendicular falls without the triangle, and both the angles given acute, therefore I substract the angle given A C B 37 degrees, 92 minutes from the angle found A C D 68 degrees 48 minutes, and their difference 30 degrees 56 minutes is the angle B C D, the second arch. Hence to finde the angle C B D, I say, as before;

As the sine of ACD 68.48. *co.ar.* 0.031382
To the sine of BCD 30.56. . 9.7 6 40
So is the co-sine of DAC 56.44. 9.742575
To the co-sine of CBD 72. 40. 9.480197

CASE 6.

Two angles and the side between them given to finde the other side.

First, by the ninth case of right angled triangles, I say, as before; As Radius, to the co-sine of the hypothenusal; so the tangent of the angle at the base, to the co-tangent of the angle at the perpendicular. Which being added to or substracted

from the other angle given, according to the direction of the fift case, giveth the second arch.

Then by the fourth Consectary of this Chapter, As the co-sine of the first angle at the perpendicular, to the co-sine of the second; so is the co-tangent of the first hypothenusal, to the co-tangent of the second.

Example.

If there were given, as in the first example of the last case, the angles B A C 56 degrees 44 minutes, and A B C 107 degrees 60 minutes, with the side A B 38 degrees, 47 minutes, to finde the side B C. The first operation will be in all things the same, and the first arch A B F 40 degrees, 28 minutes; the second arch F B C 67 degrees, 32 minutes. Hence to finde the side B C, I say:

As the co-sine of ABF 40.28.*co.ar.*0.117536
To the co-sine of FBC 67.32. 9.586119
So is the cotangent of AB 38.47.10.099862
To the co-tangent of B C 57.53 9.803517

CASE 7.

Two sides with an angle opposite to one of them, to finde the third side.

First, by the seventh case of right angled sphe-

ſphericall triangles, I ſay ; As Radius, to the co-ſine of the angle at the baſe ; ſo is the tangent of the hypothenuſal, to the tangent of the baſe, or firſt arch.

Then, by the firſt Conſectary of this Chapter, the analogie is,

As the co-ſine of the firſt hypothenuſal, to the co-ſine of the ſecond ; ſo the co-ſine of the firſt arch found, to the co-ſine of the ſecond. Which being added to or ſubſtracted from the firſt arch found, according to the direction following, their ſum or difference is the third ſide.

If the perpendicular fall

- Within the triangle, adde the firſt arch found to the ſecond arch found, and their aggregate is the ſide required.
- Without, & the angle given obtuſe, ſubſtract the firſt arch found from the ſecond arch found, and what remaineth is the third ſide.
- Without, & the given angle acute, ſubſtract the ſecond arch found from the firſt, and what remaineth is the ſide required.

1 *Example.*

In the oblique angled triangle A B C,

let there be given the sides A B 38 degrees, 47 minutes, and B C 57 degrees, 53 minutes, with the angle B A C 56 degrees, 44 minutes, and let the side A C be required. First, I let fall the perpendicular B F. and then in the right angled triangle A B F, we have given the hypothenusal A B, and the angle at the base B A F, to finde the base A F, for which I say:

As the Radius 90	10.000000
To the co-sine of B A F 56.44.	9.741576
So is the tangent of AB 38. 47.	9.900138
To the tangent of A F 23. 72.	9.641714

Secondly, for F C, I say:

As the co-sine of A B 38.47. *co.ar.*	0.106275
To the co-sine of B C 57.53.	9.729859
So is the co-sine of A F 23.72.	9.961669
To the co-sine of F C 51.12.	9.777803

Now because the perpendicular fell within the triangle, therefore I adde the first arch found A F 23 degrees, 72 minutes to the second arch found F C 51 degrees 12 minutes, and their aggregate 74 degrees, 84 minutes is *A* C the side required.

2 *Example.*

In the same triangle A B C, let there be given

given the sides AB 38 degrees, 47 minutes and AC 74 degrees 84 minutes, and the angle ABC 107 degrees, 60 minutes, and let BC be required. First then, I let fall the perpendicular AE, and continue the side BC to E, and then in the right angled triangle AEB we have given the side AB 38 degrees, 47 minutes, and the angle ABE 72 degrees, 40 minutes, the complement of ABC, to finde EB: for which I say:

As the Radius 90	10.000000
To the co-sine of ABE 72.40.	9.480538
So is the tangent of AB 38.47.	9.006138
To the tangent of EB 13.51.	9.380676

Secondly, to finde EC, I say:

As the co-sine of AB 38.47. co. ar.	0.106275
To the co-sine of AC 74.84.	9.217497
So is the co-sine of EB 13.51.	9.987813
To the co-sine of EC 71.4.	9.511585

Now because the perpendicular fall without the triangle, and the given angle obtuse, therefore I substract the first arch found EB 13 degrees 51 minutes, from the second arch EC 71 degrees, 4 minutes, and their difference 57 degrees, 53 minutes is BC, the side required.

3 *Example.*

In the same triangle *A*B C, let there be given the sides *A* C 74 degrees, 84 minutes, and B C 57 degrees, 53 minutes, and the angle B *A* C 56 deg. 44 min. to finde the side A B : I let fall the perpendicular D C, and continue the side A B to D, then in the right angled triangle A D C we have given the hypothenusal A C, and the angle at A, to finde A D.

As the Radius 90	10.000000
To the co-sine of BAC 56.44.	9.742576
So is the tangent of AC 74. 84.	10.567119
To the tangent of AD 63. 89.	10.309695

Secondly, to finde D B, I say:

As the co-sine of AC 74.84. *co. ar.*	0.582503
To the co-sine of BC 57. 53.	9.729859
So is the co-sine of AD 63. 89.	9.643545
To the co sine of DB 25.39.	9.955909

Now because the perpendicular falls without the triangle, and the angle given acute, therefore I substract the second arch found DB 25 degrees, 39 minutes, from the first arch found AD 63 degrees 89 minutes, and their difference 38 degrees 50 minutes is A B, the side required.

Case

Case 8.

Two sides with an angle opposite to one of them being given, to finde their contained angle.

First, by the ninth case of right angled spherical triangles, I say; As Radius, to the co-sine of the hypothenusal; so the tangent of the angle at the base, to the co-tangent of the angle at the perpendicular. Then, by the fourth Consectary of this Chapter, the proportion is:

As the co-tangent of the first hypothenusal, to the co-tangent of the second; so the co-sine of the first angle at the perpendicular, to the co-sine of the second: which being added to, or substracted from the first arch found, according to the direction of the seventh case, giveth the angle sought.

Example.

If there were given, as in the first example of the last case, the sides A B 38 deg. 47 min. and B C 57 deg. 53 min. with the angle B A C 56 deg. 44 min. to finde the obtuse angle A B C. The perpendicular B F falling within the triangle, then in the right angled triangle A B F, we have knowne the hypothenusal A B, and the

an-

angle at A, to finde the angle A B F, I say then,

As the Radius 90,	10.000000
Is to the co-sine of AB 38. 47.	9.893725
So is the tangent of BAF 56.44,	10.178229
To the co-tang. of ABF 40.28.	10.071954

Secondly, to finde FBC, I say:

As the co-tangent of AB 38. 47.	9.900138
To the co-tangent of BC 57.53.	9.803686
So is the co-sine of ABF 40.28.	9.882464
To the co-sine of FBC 67.32.	9.586288

Now because the perpendicular falls within the triangle, I adde the first arch found A B F 40 degrees, 28 minutes, to the second arch found F B C 67 degrees, 32 minutes, and their aggregate is 107 degr. 60 min. the angle A B C required.

Case 9.

Two angles and a side opposite to one of them being given, to finde the third angle.

First, by the ninth case of right angled sphericall triangles, I say: As the Radius, to the co-sine of the hypothenusal; so the tangent of the angle at the base, to the co-tangent of the angle at the perpendicular.

Then

Then by the third Consectary of this Chapter,, the proportion is. As the co-sine of the first angle at the base, to the co-sine of the second; so is the sine of the first angle at the perpendicular, to the sine of the second: which being added to, or substracted from the first arch found, according to the direction following, their summe or difference is the angle sought.

If the perpendicular fall	Within the triangle, adde both arches together.
	Without, and the angle opposite to the given side acute, substract the first from the second arch.
	Without, and the angle opposite to the given side obtuse, substract the second from the first.

1. *Example.*

In the oblique angled Triangle A B C, let there be given the angle B A C 56 deg. 44 min, and A C B 37 deg. 92 min. and the side A B 38 deg. 47 min, to finde the angle A B C. First, let fall the perpendicular FB, then in the right angled triangle A F B we have known, the hypothenusal A B, and the

the angle at A, to finde the angle A B F, for which I say,

As Radius, 90 deg.	10.000000
To co-sine of A B, 38.47	9.893526
So the tangent of B A F, 56.44	10.178229
To the co-tangent of ABF, 40.28.	10.071955

Secondly, to finde F B C, I say,

As the co-sine of BAF, 56.44	0.257424
To the co-sine of ACB, 37.92	9.897005
So is the sine of A B F, 40.28	9.810584
To the sine of F B C, 67.32	9.965013

Now because the perpendicular fals within the Triangle, I adde the first arch found A B F 40 deg. 28 min. to the second arch found F B C 67 deg. 32 min. and their aggregate is 107 deg. 60 min. the angle ABC required.

2. *Example.*

In the same Triangle let there be given the angle A C B 37 deg. 92 min. and A B C 107 deg. 60 min. and the side A B 38 deg. 47 min. to finde the angle B A C. First, let fall the perpendicular A E, and let the side B C be continued to E, then in the right angled triangle A E B we have known the

the Hypothenusal A B, and the angle at B, 72 deg. 40 min. the complement of A B C, to finde E A B, I say then,

As the Radius 90,	10.000000
To the co-sine of A B, 38.47	9.893726
So is the tangent of ABE, 72.40	10.498641
To the co-tangent of EAB, 22.6	10,392367

Secondly, to finde E A C, I say,

As the co-sine of ABE, 72.40	0.519462
To the co-sine of A C B, 37.92	9.897005
So is the sine of E A B, 22.6	9.574699
To the sine of E A C 78.49	9.991166

Now because the perpendicular falls without the triangle and the angle opposite to the given side acute, I substract the first angle found E A B 22 deg. 6 min. from the second arch found 78 deg. 49 min. and their difference 56 deg. 43 min. is the angle B A C required.

3 *Example.*

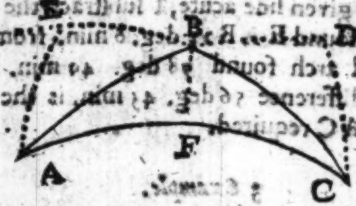
In the same triangle A B C, let there be given the angles A C B 37 deg. 92 min. and A B C 107 deg. 60 min. and the side A C 74 deg. 84 min. to finde the angle B A C. Let fall the perpendicular A E, and then in the

the right angled triangle A E C, we have known the hypothenusal A C, and the angle A C B, to finde the angle E A C.

As the Radius 90,	10.000000
To the co-sine of A C, 74.84	9.417497
So is the tangent of A C E, 37.92	9.891559
To the co-tangent of EAC, 78.49	9.309046

Secondly, to finde E A B, I say;

As the co-sine of A C E, 37.92	9.896796
To the co-sine of A B E, 17.48	9.480538
So is the sine of E A C, 78.49	9.991177
To the sine of E A B, 22.6	9.574790



Now because the perpendicular falls without the Triangle, and the angle opposite to the given side obtuse, therefore I substract the second arch found E A B, 22

deg.

deg. 6 min. from the first arch found, E A C 78 deg. 49 min. and their difference 56 deg. 43 min. is the angle B A C required.

CASE 10.

Two angles, and a side opposite to one of them being given, to finde the side between them.

First, by the 7th. Case of right angled Sphericall Triangles, I say; As Radius, to the co-sine of the angle at the base; so is the Tangent of the Hypothenusal, to the Tangent of the Base.

Then by the second Consectary of this Chapter, the proportion is, As the co-tangent of the first angle at the base, to the co-tangent of the second; so is the sine of the first base, to the sine of the second: which being added to, or substracted from, the first arch found, according to the direction of the 9th. Case, giveth the side required.

Example.

In the oblique angled triangle A B C, let there be given the two angles B A C 56 deg. 44 min. and A C B 37 deg. 92 min. with the side B C 57 deg. 53 min. to finde the side A C. Let fall the perpendicular B F, then in

in the right angled triangle B C F, we have known the Hypothenusal B C, and the angle F C B, to finde the base F C: say then,

As the Radius, 90	10.000000
Is to the co-sine of F C B, 37.92	9.897005
So is the tangent of B C, 57.53	10.196314
To the tangent of F C, 51.11	10.093319

Secondly, to finde A F, I say,

As co-tangent FCB, 37.92, *co.ar.*	9.891559
To co-tangent of B A C. 56.44	9.821771
So is the sine of F C, 51.11	9.891176
To the sine of A F, 23.72	9.604506

Now because the perpendicular falls within the Triangle, I adde the first arch F C 51 deg. 11 min. to the second arch A F, 23 deg. 72 min. and their aggregate is 74 deg. 83 min. the side A C required.

CASE II.

The three sides given to finde an angle.

The solution of this and the Case following, depends upon the Demonstration of this Proposition.

As the Rectangular figure of the sines of the sides comprehending the angle required; Is to the square of *Radius*:

So

So is the Rectangular figure of the sines of the difference of each containing side taken from the half summe of the three sides given; To the square of the sine of half the angle required.

Let the sides of the triangle ZPS be known, and let the vertical angle SZP be the angle required, then shall ZS the one be equal ZC. In like manner PS the base of the vertical angle shall be equal to PH or PB, then draw PR the sine of PZ and CK the sine of CZ or ZS. Divide CH into two equal parts in G, draw the Radius AG and let fall the perpendiculars DM and CN which are the sines of the arches PG and CG. The right line EV is the versed sine of a certain arch in a great circle, and SC the versed of the like arch in a less, then if you draw the right line NF parallel to SH bisecting CH in N, it shall also bisect the versed sine SC in F by the 15th. of the second, and RM bisecting TP in R, and drawn parallel to TX, shall for the same reason bisect PX in M, and the triangles SCH and FNC shall be like, as also the triangles TPX and RPM are like; and ZG shall be equal to the half summe of the three sides given, which thus I prove. Of any three unequal quan-

quantities given, if the difference of the two lesser be substracted from the greatest, and half the remainder added to the mean quantity, the summe shall be equall to half the summe of the three unequal quantities given.

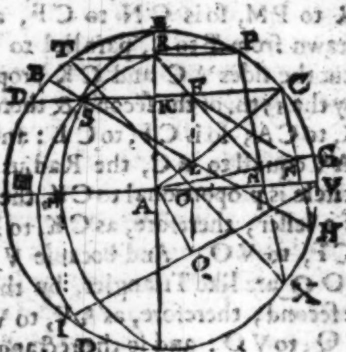
Example.

Let the quantities given be 9, 13, and 16, the difference between 9 and 13 is 4, which being substracted from 16, there remaineth 12, the half whereof is 6, which being added to 13 maketh 19, the half sum of the three unequal quantities. Now then in this Diagram P C is the difference of the two lesser sides, which taken from P H, the remainer is C H, the half whereof is C G, and C G added to C Z, the mean side, giveth G Z the half summe, and if we substract Z P the lesser containing side of the angle required, from Z G the half sum, their difference will be P G, and if we substract Z C the other side, the difference will be C G. Lastly, let the arch I V be the measure of the vertical angle P Z S, and the right line O Q bisect the lines E V and I V, and the right line A Q perpendicular to the right line I V, bisecting the same in Q, I say then.

As the Rectangular figure of the sines of the sides P R and C K, is to the square of

A C:

A C : ſo is the Rectangular figure of the ſines of the difference of each containing ſide taken from the half ſumme, that is of, P M and C N, to the ſquare of V Q the ſine of half the vertical angle S Z P, The



triangles T P X and S C H are equiangled, becauſe of their equal angles at P and C, at T and H, the angles T P X and LCH are equal, becauſe of there parallel ſides T P and S C, P H and C H, and the angles P T X and S H C are equal, becauſe the arches P G X and B P H are equal, and the double meaſure of thoſe angles, that the arches P G X and B P H are equal, may thus be proved: P B and P H are equal by conſtruction, P C and H X are equal, be-cauſe

cause of the parallel lines PX and CB, and therefore CX is equal to PB, and CP being common to both, CB must needs be equall to PX. Now then, as TP, to PX; so is CH, to CS: and as PR to PM, so is CN to CF, and a line drawn from F to L, parallel to AK, shall cut the sides AC and CK proportional by the 15th. of the second, & therefore as CK, to CA; so is CF, to CL: and because AV equall to AC, the Radius of a great circle is proportional to CK, the Radius of a lesser; therefore, as CK, to AV; so is CF, to VO. And because VAQ and VOQ are like Triangles, by the 22 of the second; therefore, as AV, to VQ; so is VQ, to VO: and so the rectangle of AV and VO is equall to the square of VQ; from which proportions this proposition may be thus deduced.

PR	Proportional	CK	And by composition	PR × CK
PM		AV		PM × VA
CN		CF		CN × CF
CF		VO		CF × VO

And dividing the two last rectangles by CF, the proportion will be

PR

PR × CK }
PM × VA }
CN }
VO }

And because VO in VA is equal to VQ square; therefore if you multiply CN by VA, the proportion will be, as PR × CK, to PM × VA; so is CN × VA, to VO × VA equal to VQ square, which was to be proved.

If then the three sides of an oblique angled spherical triangle be given, and an angle inquired; do thus:

1. Take the sines of the sides comprehending the angle inquired. Or the Logarithmes of those sines.

2. Take also the quadrat of the Radius, or the Logarithme of the Radius doubled.

3. Substract each side comprehending the angle inquired from the half sum of the three sides given, and take the sines of their differences, or the Logarithmes of those sines.

4. If the rectangle of the first divide the rectangle of the second and third, the side of the quotient is the sine of half the angle inquired.

Or if the sum of the Logarithme of the first be deducted from the sum of the Logarithmes of the second and third, the half difference is the Logarithme of half the angle sought.

Arith-

Arithmeticall illustration by Naturall Numbers.

In the Oblique angled Triangle SZP, having the
Sides PS, 42 deg. 15 min.
PZ, 30 00
And SZ, 24 7
To finde the angle PZS.

	Sines.
The side PZ, 30 deg.	50000
The side SZ, 24 deg. 7 min.	40785
1 The factus of the Sines	2039250000
2 Quadrat of the Radius	10000000000

The summe of the sides 96 deg. 22 min.
The half summe, 48 11

	Sines.
The difference of ZS 24 de. 4 min.	40737
The difference of PZ 18 11	31084
3 Factus of the sines	1266268908

Which being multiplyed by Radius square, 10000000000, and divided by 2039250000, the quotient will be 6209483277, the side whereof is 78802, the sine of 52 deg. which doubled is 104, the angle PZS inquired.

Arith-

Arithmeticall illustration by artificiall numbers.

The side P S, 42.15.	*Logar. Sine.*
The side P Z, 30	9.698970
The side S Z, 24.7	9.610503
Sum of the sides, 96.22	19.309473
The halfe sum, 48.11	
Diff. of ZS and the half sum, 24.4	9.609993
Dif. of PZ & the half sum, 18.11	9.492540
The doubled Radius	20.000000
	39.102533
From which substract the sum of the Log. of the sides, PS. PZ	19.309473
There doth remain,	19.793060

The halfe thereof, 9.896530 is the Logarithm of the sine of 52 deg. whose double 104 is the angle P Z S inquired as before.

Or if instead of the Logarithms of the sines of the sides P S and P Z, you take their Arithmeticall complements, as was shewed in the 8th. Proposition of the 4th. Chapter, and leave out the doubled Radius, the work may be performed without substraction in this manner.

The side P Z, 30	co.ar.	0.301030
The side Z S. 24.7	co.ar.	0.389497
Dif. of ZP and half sum, 18.11		9.492540
Dif. of Z S and half sum, 24.4		9.609993
The summe is		19.793060
The halfe thereof		9.896530

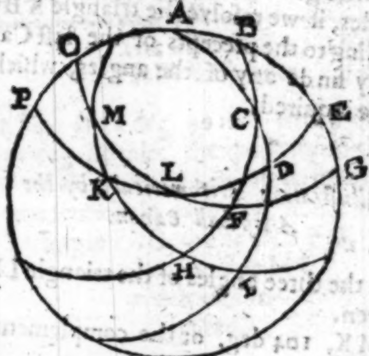
Is the Logarithm of the sine of 52 deg. as before.

Case 12.

The three angles of a Sphericall Triangle given, to finde a side.

This Case is the converse of the former, and to be resolved after the same manner, if so be we convert the angles into sides, according to the fifth of the sixth Chapter. For the two lesser angles are alwayes equal unto two sides of a Triangle comprehended by the arkes of great Circles drawn from their Poles, and the third angle may be greater then a Quadrant, and therefore the complement thereof to a Semicircle must be taken for the third side.

The angle being found, shall be one of the three sides inquired.



As in the Triangle A B C, the poles of those arks L, M, K, which connected do make the Triangle L M K, the sides of the former Triangle being equal to the angles of this latter, taking the complement of the greater angle to a semicircle for one. As A B is equal to the angle at L, or the arke E G. The side B C is equal to the angle at M, or the arch F H. And the side A C is equal to the complement of the angle LKM, or the arch DI. Therefore if the angles of the latter triangle L M K be given, the sides of the former triangle A B, B C, and A C are likewise given. And the angles

 of

of the triangle L M K being thus converted into sides, if we resolve the triangle A B C, according to the precepts of the last Case, we may finde any of the angles, which is the side inquired.

Illustration Arithmetical, by the Artificiall Canon.

Let the three angles of the triangle LMK be given.

L M K, 104 deg. or the complement of D K I, 76 deg. equal to A C.

M L K, or the side A B, 46 deg. 30 min.

L M K, or the side B C, 36 deg. 14 min.

To finde the side M L, or the angle A B C.

The sides	A C	76.	
	A B	46.30	9.859118
	B C	36.14	9.770675
Sum of the sides		158.44	19.629893
Halfe sum		79.22	
Diff. of A B and the sum		32.92	9.735573
Dif. of B C and half sum		43.08	9.834432
The doubled Radius			20.000000

The sum

	The ſumme	39.569605
Sum of the ſides	*ſubſtract*	19.629893
	The difference	19.939712
	Halfe difference	9.969856

The Sine of 68 deg. 90 min. which doubled is 137 deg. 80 min. the quantity of the angle A B C, and the complement thereof to a ſemicircle 42 deg. 20 min. is the angle FBG, or the arch F G, equal to the ſide M L which was inquired.

The [illegible]	[illegible]
sum of Quadrats [illegible]	10.[illegible]
The difference	10.95912
Is the difference	9.[illegible]

The Sine of [illegible] which doubled is [illegible] deg. [illegible], the quantity of the angle ABC, and the complement thereof to a semicircle 41 deg. [illegible] is the angle FBG, or the arch FG, equal to the side ML which was inquired.

M 3

Institutio Mathematica: OR, A MATHEMATICALL Institution:

The second Part.

Containing the application and use of the Naturall and Artificiall *SINES* and *TANGENTS*, as also of the *LOGARITHMS*, IN

Astronomie,
Dialling, *and*
Navigation.

By *JOHN NEWTON*.

LONDON,
Printed *Anno Domini*, 1654.

A

Mathematicall Institution :

The second Part.

CHAP. I.

Of the Tables of the Suns motion, and of the equation of time for the difference of Meridians.

WHereas it is requisite that the Reader should be acquainted with the Sphere, before he enter upon the practise of *Spherical Trigonometri*, the which is fully explained in *Blundeviles* Exercises, or *Chilmades* translation of *Hues* on the Globes, to whom I refer those that are not yet acquainted therewith: that which I here intend is to show the use of *Trigonometrie* in the actuall resolution of some known Triangles of the Sphere.

And because the Suns place or distance from the next Equinoctial point is usually one of the three terms given in Astronomical Questions, I will first shew how to compute that by Tables calculated in Decimal numbers according to the Hypothesis of *Bullialdus*, and for the Meridian of *London*, whose Longitude reckoned from the *Canarie* or *Fortunate* Islands is 21 deg. and the Latitude, North, 51 deg. 57 parts (min.) or centesms of a degree.

Nor are these Tables so confined to this Meridian, but that they may be reduced to any other: If the place be East of *London*, adde to the time given, but if it be West make substraction, according to the difference of Longitude, allowing 15 deg. for an houre, and 6 minutes or centesms of an houre to one degree, so will the sum or difference be the time æquated to the Meridian of *London*, and for the more speedy effecting of the said Reduction, I have added a Catalogue of many of the chiefest Towns and Cities in diverse Regions, with their Latitudes and difference of Meridians from *London* in time, together with the notes of Addition and Substraction, the use whereof is thus.

Suppose

Suppose the time of the Suns enterance into *Taurus* were at *London* Aprill the 10th. 1654, at 11 of the clock and 16 centesms before noon, and it be required to reduce the same to the Meridian of *Uraniburge*, I therefore seeke *Uraniburge* in the Catalogue of Cities and Places, against which I finde 83 with the letter A annexed, therefore I conclude, that the Sun did that day at *Uraniburge* enter into *Taurus* at 11 of the clock and 99 min. or centesms before noon, and so of any other.

Problem I.

To calculate the Suns true place.

THe form of these our Tables of the Suns motion is this, In the first page is had his motion in *Julian* years compleat, the *Epochaes* or roots of motions being prefixed, which sheweth the place of the Sun at that time where the *Epocha* adscribed hath its beginning: the Tables in the following pages serve for *Julian* Years, Moneths, Dayes, Houres, and Parts, as by their Titles it doth appear. The Years, Moneths, and Dayes, are taken compleat, the Houres and Scruples current. After these Tables followeth another, which contains

tains the *Æquations* of the Eccentrick to every degree of a Semicircle, by which you may thus compute the Suns place.

First, Write out the *Epocha* next going before the given time, then severally set under those the motions belonging to the years, moneths, and dayes compleat, and to the hours and scruples current, every one under his like, (onely remember that in the Bissextile year, after the end of *February*, the dayes must be increased by an unit) then adding them all together, the summe shall be the Suns mean motion for the time given.

Example.

Let the given time be 1654, *May* 13, 11 hours, 25 scruples before noon at *London*, and the Suns place to be sought.

The

The numbers are thus:

		Longit. ☉		Aphel. ☉	
The Epocha	1640	291.	2536	96.	2297
Years compl.	13	359.	8508		2052
Moneth co. April		118.	3775		53
Dayes compl.	12	11.	8278		6
Hours	23		9444		
Scruples	25		102		
Sũ or mean motiõ		782.	1643	96.	4408

2. Substract the Aphelium from the mean Longitude, there rests the mean Anomalie; if it exceed not 360 degrees, but if it exceed 360 degr. 360 being taken from their difference, as oft as it can, the rest is the mean Anomalie sought.

Example.

The ☉ mean Longitude	782 . 1643
The Aphelium substracted	96 . 4308
There rests	685 . 7335
From whence deduct	360 .
There rests	325 . 7335
the mean Anomalie.	

3. With the mean Anomalie enter the Table of the Suns Eccentrick Equation, with-

with the degree descending on the left side, if the number thereof be lesse then 180; and ascending on the right side, if it exceed 180, and in a straight line you have the Equation answering thereunto, using the part proportional, if need require.

Lastly, according to the title Add or Substract this Equation found to or from the mean longitude; so have you the Suns true place.

Example.

The Suns mean longitude	782. 1643
Or deducting two circles,	720.
The Suns mean longitude is	62. 1643
The Suns mean Anomalie	325. 7335

In this Table the Equation answering to 325 degrees is 1. 1525
The Equation answering to 326 degrees is 1. 1236
And their difference 289.

Now then if one degree or 10000
Give 289
What shall 7335
Give, the product of the second and third term is 2119815, and this divided by 10000 the first term given, the quotient or term required

quired will be 212 *fere*, which being deducted from 1.1525, the Equation answering to 325 degr. because the Equation decreased, their difference 1.1313. is the true Equation of this mean Anomalie, which being added to the Suns mean longitude, their aggregate is the Suns place required.

Example.

The Suns mean longitude	62 . 1643
Equation corrected Add	1 . 1313
The Suns true place or Longitude	63 . 2956

That is, 2 Signes, 3 degrees, 29 minutes, 56 parts.

The Suns Equation in this example corrected by Multiplication and Division may more readily be performed by Addition and Substraction with the help of the Table of Logarithmes: for,

As one degree, or 10000,	4.000000
Is to 289;	2.460898
So is 7335,	3.865400
To 212 *fere*	2.326298

The

The Suns mean Motions.

Epochæ	Longitud ☉ °	′	″	Aphelium ☉ °	′	″
Per. Jul.	242	99	61	355	85	44
Mundi	248	71	08	007	92	42
Christi	278	98	69	010	31	36
1600	290	95	44	095	58	78
1620	291	10	41	095	90	39
1640	291	25	36	096	21	97
1660	291	40	33	096	53	56
1	359	76	11	0	01	58
2	359	52	22	0	18	17
3	359	28	30	0	04	74
B 4	000	03	00	0	06	90
5	359	79	11	0	07	89
6	359	55	19	0	09	47
7	359	31	30	0	11	05
B 8	000	05	97	0	12	64
9	359	82	08	0	14	22
10	359	58	19	0	15	78
11	359	34	30	0	17	36
B 12	000	08	97	0	18	94
13	359	85	08	0	20	52
14	359	00	19	0	22	11
15	359	37	30	0	23	69
B 16	000	11	97	0	25	25
17	359	88	08	0	26	83
18	359	64	19	0	28	41
19	359	40	28	0	30	00
B 20	000	14	97	0	31	61
40	000	29	91	0	63	19
60	000	44	83	0	94	77
80	000	59	83	1	26	39
100	000	74	80	1	57	97

The Suns mean Motions.

	Longitude ☉			*Aphelium* ☉		
	°	′	″	°	′	″
100	00	74	80	01	57	97
200	01	49	58	03	15	94
300	02	24	39	04	73	94
400	02	99	19	06	31	94
500	03	73	97	07	89	91
600	04	48	77	09	47	92
700	05	23	58	11	05	89
800	05	98	36	12	63	89
900	06	73	17	14	21	86
1000	07	47	97	15	79	86
2000	14	95	92	31	59	69
3000	22	43	89	47	39	55
4000	29	91	82	63	19	41
5000	37	39	80	78	99	25
January	030	55	50	0	00	14
February	058	15	30	0	00	25
March	088	70	83	0	00	39
April	118	27	75	0	00	53
May	148	83	28	0	00	67
June	178	40	19	0	00	80
July	208	95	69	0	00	94
August	239	51	22	0	01	06
September	269	08	17	0	01	19
October	299	63	66	0	01	33
November	329	20	61	0	01	44
December	359	76	11	0	01	58

The Suns mean motions in Dayes.

D	*Longit.* ☉ °	'	''	*Aphel.* '	''
1	0	98	55	0	00
2	1	97	14	0	00
3	2	95	69	0	00
4	3	94	25	0	02
5	4	92	83	0	02
6	5	91	39	0	03
7	6	89	94	0	03
8	7	88	52	0	03
9	8	87	08	0	05
10	9	85	63	0	05
11	10	84	22	0	05
12	11	82	78	0	06
13	12	81	33	0	06
14	13	79	91	0	06
15	14	78	47	0	06
16	15	77	03	0	08
17	16	75	61	0	08
18	17	74	16	0	08
19	18	72	72	0	08
20	19	71	30	0	08
21	20	69	86	0	08
22	21	68	41	0	11
23	22	67	00	0	11
24	23	65	56	0	11
25	24	64	11	0	11
26	25	62	94	0	11
27	26	61	25	0	11
28	27	59	80	0	11
29	28	58	36	0	13
30	29	56	94	0	14
31	30	55	50	0	14
32	31	54	05	0	14

H	Longit. ☉			M	Long.		M	Long.	
	°	'	"		'	"		'	"
1	0	04	11	34	1	39	67	2	75
2	0	08	22	35	1	43	68	2	79
3	0	12	31	36	1	47	69	2	83
4	0	16	42	37	1	51	70	2	87
5	0	20	52	38	1	56	71	2	91
6	0	24	63	39	1	60	72	2	96
7	0	28	75	40	1	64	73	3	00
8	0	32	86	41	1	68	74	3	04
9	0	36	97	42	1	72	75	3	08
10	0	41	06	43	1	76	76	3	12
11	0	45	17	44	1	80	77	3	16
12	0	49	27	45	1	84	78	3	20
13	0	53	39	46	1	88	79	3	24
14	0	52	50	47	1	93	80	3	28
15	0	61	61	48	1	97	81	3	32
16	0	65	72	49	2	01	82	3	37
17	0	69	80	50	2	05	83	3	41
18	0	73	91	51	2	09	84	3	45
19	0	78	03	52	2	13	85	3	49
20	0	82	14	53	2	17	86	3	53
21	0	86	25	54	2	21	87	3	57
22	0	90	36	55	2	25	88	3	61
23	0	94	44	56	2	30	89	3	65
24	0	98	55	57	2	34	90	3	69
25	1	02	66	58	2	38	91	3	74
26	1	06	77	59	2	42	92	3	78
27	1	10	88	60	2	46	93	3	82
28	1	14	99	61	2	50	94	3	86
29	1	19	10	62	2	54	95	3	90
30	1	23	21	63	2	58	96	3	94
31	1	27	32	64	2	62	97	3	98
32	1	31	43	65	2	67	98	4	02
33	1	35	54	66	2	71	99	4	06
'	'	"	‴	'	'	"	100	4	11
"	"	‴	⁗	"	"	‴	"	"	‴

The Equations of the Suns Eccentrick.

	Æq. sub					Æq. sub			
	°	′	″			°	′	″	
0	0	00	00	360	30	1	00	19	330
1	0	03	52	359	31	1	03	33	329
2	0	07	03	358	32	1	06	41	328
3	0	10	56	357	33	1	09	41	327
4	0	14	05	356	34	1	12	36	326
5	0	17	13	355	35	1	15	25	325
6	0	21	00	354	36	1	18	03	324
7	0	24	44	353	37	1	20	78	323
8	0	27	89	352	38	1	23	50	322
9	0	31	30	351	39	1	26	22	321
10	0	34	72	350	40	1	28	91	320
11	0	38	17	349	41	1	31	58	319
12	0	41	56	348	42	1	34	22	318
13	0	44	94	347	43	1	36	86	317
14	0	48	30	346	44	1	39	50	316
15	0	51	67	345	45	1	41	78	315
16	0	55	03	344	46	1	44	52	314
17	0	58	36	343	47	1	47	05	313
18	0	61	67	342	48	1	49	47	312
19	0	64	97	341	49	1	51	89	311
20	0	68	24	340	50	1	54	16	310
21	0	71	53	339	51	1	56	47	309
22	0	74	78	338	52	1	58	69	308
23	0	78	03	337	53	1	60	86	307
24	0	81	22	336	54	1	63	00	306
25	0	84	41	335	55	1	65	14	305
26	0	87	56	334	56	1	67	25	304
27	0	90	69	333	57	1	69	30	303
28	0	94	26	332	58	1	71	33	302
29	0	97	32	331	59	1	73	28	301
30	1	00	19	330	60	1	75	05	300
		Add.					Add.		

The Equations of the Suns Eccentrick.

	Æq. Sub			
	°	′	″	
60	1	75	05	300
61	1	76	92	299
62	1	76	69	298
63	1	80	39	297
64	1	81	97	296
65	1	83	50	295
66	1	85	00	294
67	1	86	44	293
68	1	87	83	292
69	1	89	16	291
70	1	90	44	290
71	1	91	69	289
72	1	92	86	288
73	1	93	96	287
74	1	95	28	286
75	1	96	22	285
76	1	97	14	284
77	1	97	97	283
78	1	98	72	282
79	1	99	61	281
80	2	00	41	280
81	2	01	14	279
82	2	01	72	278
83	2	02	25	277
84	2	02	94	276
85	2	03	14	275
86	2	03	44	274
87	2	03	66	273
88	2	04	05	272
89	2	04	22	271
90	2	04	41	270
	Add.			

	Æq. Sub			
	°	′	″	
90	2	04	41	270
91	2	04	47	269
92	2	04	41	268
93	2	04	27	267
94	2	04	11	266
95	2	03	89	265
96	2	03	61	264
97	2	03	33	263
98	2	02	94	262
99	2	02	50	261
100	2	02	03	260
101	2	01	42	259
102	2	00	64	258
103	1	99	83	257
104	1	99	27	256
105	1	98	47	255
106	1	97	64	254
107	1	96	67	253
108	1	95	67	252
109	1	94	55	251
110	1	93	39	250
111	1	92	11	249
112	1	90	89	248
113	1	89	58	247
114	1	88	28	246
115	1	86	89	245
116	1	85	44	244
117	1	83	97	243
118	1	82	39	242
119	1	80	72	241
120	1	79	02	240
	Add.			

The Equations of the Suns Eccentrick.

	Æq. Sub					Æq. Sub			
	°	′	″			°	′	″	
120	1	79	00	240	150	1	04	27	210
121	1	77	19	239	151	1	01	00	209
122	1	75	39	238	152	0	97	75	208
123	1	73	50	237	153	0	94	47	207
124	1	71	50	236	154	0	91	19	206
125	1	69	50	235	155	0	87	89	205
126	1	67	53	234	156	0	84	58	204
127	1	65	39	233	157	0	81	28	203
128	1	63	22	232	158	0	77	97	202
129	1	61	28	231	159	0	74	61	201
130	1	58	77	230	160	0	71	25	200
131	1	56	44	229	161	0	67	86	199
132	1	54	05	228	162	0	64	44	198
133	1	51	64	227	163	0	60	97	197
134	1	49	16	226	164	0	57	44	196
135	1	46	97	225	165	0	53	89	195
136	1	44	16	224	166	0	50	33	194
137	1	41	58	223	167	0	46	75	193
138	1	38	94	222	168	0	43	19	192
139	1	36	31	221	169	0	39	64	191
140	1	33	58	220	170	0	36	06	190
141	1	30	83	219	171	0	32	50	189
142	1	28	08	218	172	0	28	91	188
143	1	25	28	217	173	0	25	31	187
144	1	22	42	216	174	0	21	69	186
145	1	19	55	215	175	0	17	08	185
146	1	16	67	214	176	0	14	47	184
147	1	13	72	213	177	0	10	86	183
148	1	10	61	212	178	0	07	25	182
149	1	07	47	211	179	0	03	64	181
150	1	04	27	210	180	0	00	00	180
	Add.					Add.			

A Catalogue of ſome of the moſt eminent Cities and Towns in *England*, *Ireland*, and other Countreys, wherein is ſhewed the difference of their Merdians from *London*, with the height of the Pole Artique.

Names of the Places.		Diff. in time		Pole	
A*Berden* in *Scotland*	S	0	12	58	67
S. *Albons*	S	0	02	51	92
Alexandria in Egypt	A	2	18	30	97
Amſterdam in *Holland*	A	0	35	52	42
Athens in *Greece*	A	1	87	37	70
Bethelem	A	2	77	31	85
Barwick	S	0	10	55	82
Bedford	S	0	03	52	30
Calice in *France*		0	00	50	87
Cambridge	A	0	03	52	33
Canterbury	A	0	08	51	45
Conſtantinople	A	2	30	43	00
Darby	S	0	08	53	10
Dublin in *Ireland*	S	0	43	53	18
Dartmouth	S	0	25	50	53
Ely	A	0	02	52	33
Grantham	S	0	03	52	97

Gla-

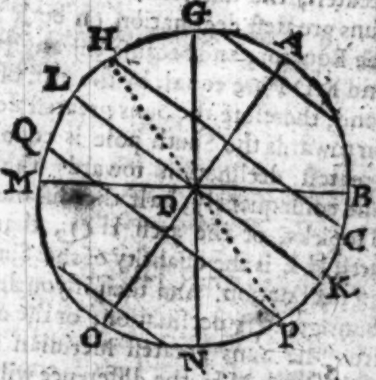
Glocester	S	[illegible]	[illegible]	52	00
Hartford	S	0	02	51	83
Hierusalem	A	3	08	32	17
Huntington	S	0	02	52	32
Leicester	S	0	07	52	67
Lincolne	S	0	02	57	25
Nottingham	S	0	07	53	05
Newark	S	0	05	53	03
Newcastle	S	0	10	54	97
Northampton	S	0	07	52	30
Oxford	S	0	08	51	90
Peterborough	S	0	03	52	38
Richmond	S	0	10	54	43
Rochester	A	0	05	51	47
Rochel in *France*	S	0	07	45	81
Rome in *Italy*	A	0	83	42	03
Stafford	S	0	13	52	92
Stamford	S	0	03	52	68
Shrewsbury	S	0	18	54	80
Tredagh in *Ireland*	S	0	45	53	63
Uppingham	S	0	05	52	67
Uraniburge	A	0	83	55	90
Warwick	S	0	10	52	42
Winchester	S	0	08	51	17
Waterford in *Ireland*	S	0	45	52	37
Worcester	S	0	15	52	33
Yarmouth	A	0	10	52	75
York	S	0	07	54	00
LONDON		0	00	51	53

Probl.

Probl. 2.

To finde the Suns greatest declination, and the Poles elevation.

THe Declination of a Planet or other Star is his distance from the Equator, and as he declines from thence either Northward or Southward, so is the Declination thereof counted either North or South.



In the annexed Diagram, G M N B represents the Meridian, L K the Equinoctiall, H P the Zodiac, A the North pole, O the South, M B the Horizon, G the Zenith, N

N the

the Nadir, H C a parallel of the Suns diurnall motion at H, or the Suns greatest declination from the Equator towards the North pole, P Q a parallel of the Suns greatest declination from the Equator towards the South pole. From whence it is apparent, that from M to H is the Suns greatest Meridian altitude, from M to Q his least; if therefore you deduct M Q, the least Meridian altitude from M H, the greatest, the difference will be H Q, the Suns greatest declination on both sides of the Equator, and because the angles H D L and K D P are equal, by the 9th. of the second, therefore the Suns greatest declination towards the South pole is equall to his greatest declination towards the North; and consequently, half the distance of the Tropicks, or the arch H Q, that is, the arch H L is the quantity of the Suns greatest declination. And then if you deduct the Suns greatest declination, or the arch H L from the Suns greatest Meridian altitude, or the arch M H, the difference will be ML, or the height of the Equator above the Horizon, the complement whereof to a Quadrant is the arch M O equal to A B, the height of the Pole.

Examp

Example.

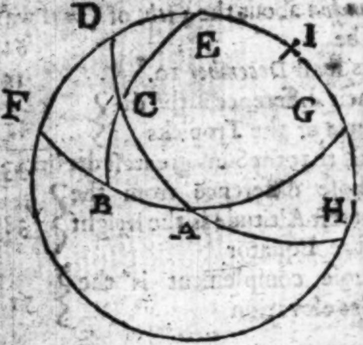
The Suns greatest meridian altitude at *London* about the 11th. of *June* was found to be - 62 00 00
His least *December* 10. 14 54 00
Their difference is the distance of the Tropicks } 47 06 00
Half that the Suns greatest declin. 23 53 00
Whose difference from the greatest Altitude is the height of the Equator } 38 47 00
Whose complement is the Poles elevation } 51 53 00

Probl. 3.

The Suns place and greatest declination given to finde the declination of any point of the Ecliptique.

IN this figure let DFHG denote the Solsticiall Colure, F B A G the Equator, D A H the Ecliptique, I the Pole of the Ecliptique, E the Pole of the Equator, CEB a Meridian line passing from E through the Sun at C, and falling upon the Equator F A G with right angles in the point B. Then is D A F the angle of the Suns greatest declination, A C the Suns distance from *Aries*

the next Equinoctiall point, B C the declination of the point sought.



Now suppose the sun to be in 00 deg. of *Gemini*, which point is distant from the next Equinoctiall point 60 deg. and his declination be required. In the rectangled sphericall triangle we have known, 1 The hypothenusal A C 60 deg. 2 The angle at the base B A C 23 deg. 53 min. Hence to finde the perpendicular B C, by the 8 Case of right angled sphericall triangles, the analogie is,

As the Radius, 90	10.000000
To the sine of B A C, 23.53.	9.601222
So is the sine of A C, 60	9.937531
To the sine of B C, 20,22	9.538753

Probl. 4.

The greatest declination of the Sun, and his distance from the next Equinoctial point given, to finde his right ascension.

IN the Triangle A B C of the former diagram, having as before, the angle BAC, and the hypothenusal AC, the Right Ascension of the sun AB may be found by the 7 Case of right angled spherical triangles: for

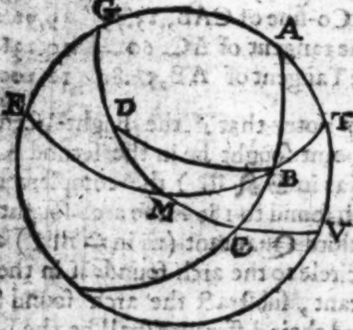
As the Radius, 90	10.000000
To the Co-sine of CAB, 23.53	9.962299
So is the tangent of AC, 60	10.238561
To the Tangent of AB, 57.80	10.200860

Only note, that if the Right Ascension of the point sought be in the second Quadrant (as in ♋ ♌ ♍) the complement of the arch found to 180 is the arch sought. If in the third Quadrant (as in ♎ ♏ ♐) adde a semicircle to the arch found; if in the last Quadrant, substract the arch found from 360, and their difference shall be the Right Ascension sought.

Probl. 5.

The Latitude of the place, and declination of the Sun given, to finde the Ascensionall difference, or time of the Suns rising before or after the houre of six.

THe Ascensionall difference is nothing else but the difference between the Ascension of any point in the Ecliptique in a right Sphere, and the ascension of the same point in an oblique Sphere.



As in the annexed Diagram, A G E V represents the Meridian, E M T the Horizon, G M C V

GMCV the Equator, A the North Pole, VT the complement of the Poles elevation, BC the Suns declination, DB an arch of the Eclipique, DC the Right Ascension, MC the Ascensionall difference. Then in the right angled triangle BMC, we have limited,

1 The angle BMC, the complement of the Poles elevation, 38 deg. 47 min.

2 The perpendicular BC, the Suns Declination 20 deg. 22 min.

Hence to finde MC the Ascensional difference, By the 6 Case of right angled Spherical Triangles, the Proportion is,

As the Radius, 90	10.000000
To the tangent of BC, 20.22	9.566231
So is co-tangent of BMC, 38.47,	10.099861
To the sine of MC, 27.62	9.666092

Probl. 6.

The Latitude of the place, and the Suns Declination given, to finde his Amplitude.

THe Suns Amplitude is an arch of the Horizon intercepted between the Equator, and the point of rising, that is, in the preceding Diagram the arch MB,

therefore in the right angled Sphericall triangle M B C, having the angle B M C the height of the Equator, 38 deg. 47 min. and B C the Suns declination 20 de. 22 m. given, the hypothenusal M B may be found by the 5 Case of right angled sphericall triangles: for

As the sine of B M C, 38,47	9.793863
Is to the Radius, 90.	10.000000
So is the sine of B C, 20.22	9.538606
To the sine of M B, 33.75	9.744743

Probl. 7.

The Latitude of the place, and the Suns Declination given, to finde the time when he will be East or west.

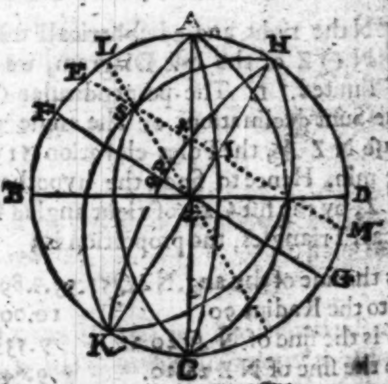
LEt A B C D in the annexed diagram represent the Meridian, B D the Horizon, F G the Equator, H N K an arch of a Meridian, A C the Azimuth of East and West, or first Verticall, E M, a parallel of declination. Then in the right angled sphericall triangle A H N, we have known,

1 The perpendicular A H, the complement of the Poles elevation, 38 deg. 47 mi.

2 The

2 The hypothenusal H N, the complement of the Suns declination, 69 deg. 78 m.

Hence the angle A H N may be found by the 13 Case of right angled sphericall triangles.



As the Radius 90	10,000000
To the tangent of A H 38. 47.	9.900138
So is the co-tangent H N 69.78.	9.566231
To the co-sine of A H N 72. 98.	9.466369

Whose complement N H Z 17 degr. 2 min. being converted into time, giveth one houre, 13 minutes, or centesmes of an hour, and so much is it after six in the morning when the Sun will be due East, and before six at night, when he will be due West.

Probl. 8.

The Latitude of the place and Declination of the Sun given, to finde his Altitude when he cometh to be due East or west.

IN the right angled sphericall triangle N Q Z of the last Diagram, we have limited. 1. The perpendicular Q N, the Suns declination. 2. The angle at the base N Z Q, the Poles elevation 51 degr. 53 min. Hence to finde the hypothenusal N Z, by the fift Case of right angled sphericall Triangles, the proportion is;

As the sine of the ang. N Z Q 51.53.	9.893725
Is to the Radius 90	10.000000
So is the sine of N Q 20.22.	9.538606
To the sine of N Z 26.20.	9.644881

Probl. 9.

The Latitude of the place, and Declination of the Sun given, to fine the Suns Azimuth at the hour of six.

IN the right angled sphericall triangle A I H of the seventh Probleme, we have known: 1. The base A H, the complement of the Poles elevation 38 degr. 47 min.

min. and the perpendicular I H, the complement of the Suns declination 69 degr. 78 min. Hence to finde the angle at the base H A I the suns Azimuth at the houre of six, by the 11 Case of right angled spherical triangles, the proportion is,

As the Radius, 90	10.000000
To the sine of A H, 38.47	9.793863
So the co-tangent of H I, 69.78	9.566231
To the co-tangent of H A I, 77	9.360094

Probl. 10.

The Poles elevation, with the Suns Altitude and Declination given, to finde the Suns Azimuth.

IN the oblique angled Spherical triangle A H S, in the Diagram of the seventh Probleme, we have known, the side A H, the complement of the Poles elevation, 38 deg. 47 min. H S, the complement of the Suns declination, 74 deg. 83 min. And the side S A, the complement of the Suns altitude, 57 deg. 53 min, to finde the angle S A H: Now then, by the 1 Case of Oblique angled Sphericall Triangles, I work as is there directed.

SH,	[illegible].83	
HA,	38,47	9.793863
SA,	57.53	9.926174
Summe of the sides	170.83	19.720037
Halfe summe	85.41.50	
Dif. of HA & half sum,	46.94.50	9,863737
Dif. of SA & half sum,	27.88.50	9.669990
The doubled Radius		20.000000
Their summe		39.533727
From whence substract		19.720037
There rests		19.813690
The halfe whereof		9.906845

Is the sine of 53 deg. 80 min. which doubled is 107 deg. 60 min. the Suns Azimuth from the north, and 72 deg. 40 min. the complement thereof to a Semicircle is the Suns Azimuth from the South.

CHAP.

CHAP. II.

THE ART OF *SHADOWS*:

Commonly called

DIALLING.

Plainly shewing out of the Sphere, the true ground and reason of making all kinde of Dials that any plain is capable of.

Problem 1.

How to divide diverse lines, and make a Chord to any proportion given.

Forasmuch as there is continuall use both of Scales and Chords in drawing the Scheams and Dials following, it will be necessary first to shew the making of them, that such

as

as cannot have the benefit of the skilful artificers labour, may by their own pains supply that defect.

Draw therefore upon a piece of paper or pastboard a streight line of what length you please, divide this line into 10 equal parts, and each 10 into 10 more, so is your line divided into 100 equal parts, by help where of a line of Chords to any proportion may be thus made.

First, prepare a Table, therein set down the degrees, halves, and quarters, if you please, from one to 90. Unto each degree and part of a degree joyn the Chord proper to it, which is the naturall sine of halfe the arch doubled, by the 19th. of the second of the first part: if you double then the naturall sines of 5. 10. 20. 30. degrees, you shall produce the Chords of 10. 20. 40. 60. degrees: Thus 17364 the sine of 10 de. being doubled, the sum will be 34628, the Chord of 20 deg. and so of the rest as in the Table following.

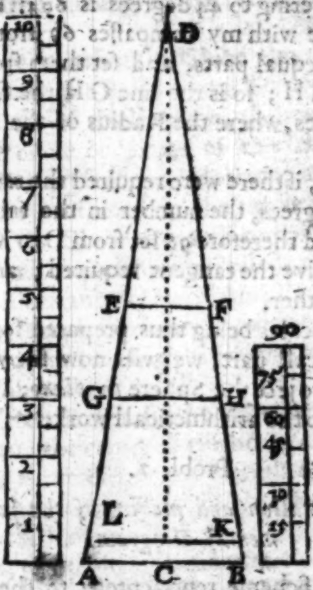
De	Chord	De	Chord	De	Chord
1	17	31	534.	61	1015
2	35	32	551	62	1030
3	52	33	568	63	1045
4	70	34	585	64	1060
5	87	35	601	65	1074
6	105	36	618	66	1089
7	122	37	635	67	1104
8	139	38	651	68	1118
9	157	39	668	69	1133
10	175	40	684	79	1147
11	192	41	700	71	1161
12	209	42	717	72	1176
13	226	43	733	73	1190
14	244	44	749	74	1204
15	261	45	765	75	1217
16	278	44	781	76	1231
17	296	47	797	77	1245
18	313	48	813	78	1259
19	330	49	830	79	1273
20	347	50	845	80	1286
21	364	51	861	81	1299
22	382	52	876	82	1312
23	398	53	892	83	1325
24	416	54	908	84	1338
25	432	55	923	85	1351
26	450	56	939	86	1364
27	466	57	954	87	1377
28	384	58	970	88	1389
29	501	59	984	89	1402
30	318	60	1000	90	1414

This done, proportion the Radius of a circle to what extent you pleaſe, make A B equal thereto, in the middle whereof, as in C, erect the perpendicular C D, and draw the lines A D and B D, equal in length to your line of equal parts, ſo have you made an equiangled Triangle, by help whereof and the Table aforeſaid, the Chord of any arch proportionable to this Radius may ſpeedily be obtained.

As for example. Let there be required the Chord of 30 deg. the number in the Table anſwering to this arke is 518, or in proportion to this Scale 52 almoſt, I take therefore 52 from the Scale of equal parts, and ſet them from D to E and F, and draw the line E F, which is the Chord deſired. Thus may you finde the Chord of any other arch agreeable to this Radius. Or if your Radius be either of a greater or leſſer extent, if you make the baſe of your Triangle A B equal thereunto, you may in like manner finde the Chord of any arch agreeable to any Radius given. Only remember that if the Chord of the arch deſired exceed 60 deg. the ſides of the Triangle A D and D B muſt be continued from A and B as far as need ſhall require. In this manner is made the line of Chords adjoyning, an-

ſwerable

swerable to the Radius of the Fundamental Scheme.



And in this manner may you finde the Sine, Tangent or Secant of any arch proportionable to any Radius, by help of the Canon of Naturall Sines, Tangents and Secants, and the aforesaid Scale of equall parts, as by example may more plainly appear.

Let

Let there be required the sine of 44 degrees in the table of natural sines, the number answering to 44 degrees is 694. I take therefore with my compasses 69 from my Scale of equal parts, and set them from D to G and H; so is the line G H the sine of 44 degrees, where the Radius of the circle is A B.

Again, if there were required the tangent of 44 degrees, the number in the table is 965; and therefore 96 set from D to K and L shall give the tangent required; and so for any other.

Your Scales being thus prepared for the Mechanicall part, we will now shew you how to project the Sphere *in plano*, and so proceed to the arithmeticall work.

Probl. 2.

The explanation and making of the fundamental Diagram.

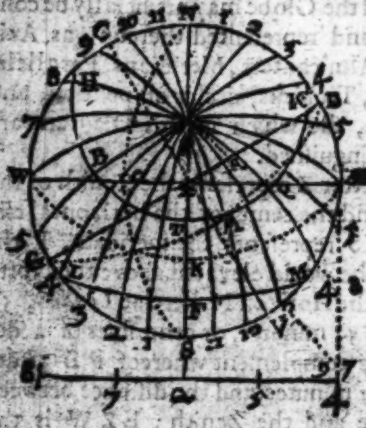
THis Scheme representeth to the eye the true and natural situation of those circles of the Sphere, whereof we shall have use in the description of such sorts of Dials as any flat or plane is capable of. It is therefore necessary first to explain that, and the making thereof, that the Symetry

metry of the Scheme with the Globe being well understood, the representation of every plane therein may be the better conceived.

Suppose then that the Globe elevated to the height of the Pole be prest flat down into the plane of the Horizon, then will the outward circle or limbe of this Scheme N E S W represent that Horizon, and all the circles contained in the upper Hemisphere of the Globe may artificially be contrived, and represented thereon, as Azimuths, Almicanters, Meridians, Parallels, Equator, Tropicks, circles of position, and such like, the which in this Diagram are thus distinguished.

The letter Z represents the Zenith of the place, and the center of the horizontal circle N Z S represents the meridian, P the pole of the world elevated above the North part of the Horizon N here at *London*, 51 degrees, 53 minutes, or centesimes of a degree, the complement whereof P Z 38 degrees, 47 minutes, and the distance between the Pole and the Zenith; E Z W is the prime vertical, D Z G and C Z V any other intermediate Azimuths, N O S a circle of position, E K W the Equator, the distance whereof from Z is equal to P N, the height of

of the Pole, or from S equal to P Z, the complement thereof, H B Q X the Tropick or parallel of *Cancer*, L F M, the Tropique of *Capricorn*, the rest of the circles intersecting each other in the point P, are the meridians or hour-circles, cutting the Horizon and other circles of this Diagram in such manner as they do in the Globe it self.



Amongst these the Azimuths onely in this projection become streight lines, all the rest remain circles, and are greater or lesser, according to their natural situation in

in the Globe, and may be thus described.

Open your compasses to the extent of the line A B in the former Probleme, (or to any other extent you please) with that Radius, or Semidiameter describe the horizontal circle N E S W, crosse it at right angles in Z with the lines N Z S and E Z W.

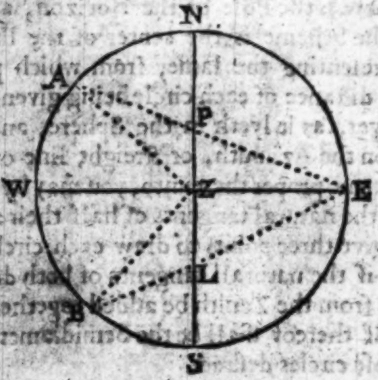
That done, seek the place of the Pole at P, through which the hour circles must pass, the Equinoctial point at K, the Tropiques at T and F, the reclining circle at O, and the declining reclining at A; all which may thus be found.

The Zenith in the Globe or Materiall Sphere is the Pole of the Horizon, and Z in the Scheme is the center of the limbe, representing the same, from which point the distance of each circle being given both wayes, as it lyeth in the Sphere, and set upon the Azimuth, or streight line of the Scheme proper thereunto, you may by help of the natural tangents of half their arches give three points to draw each circle by, for if the naturall tangents of both distances from the Zenith be added together, the half thereof shall be the Semidiameters of those circles desired.

The reason why the natural tangent of half the arches are here taken, may be

made

made plain by this Diagram following. Wherein making E Z the Radius, S Z N is a tangent line thereunto, upon which if you will project the whole Semicircle S W N, it is manifest, by the work, that every part of the lines Z N or Z S can be no more then the tangent of half the arch desired, because the whole line Z N or Z S is the tang. of no more then half the Quadrant, that is, of 45 degrees, by the 19th. of the second Chapter of the first Part; and therefore W E A is but half the angle W Z A and W E B is but half the angle W Z B.



Now then if E Z or Radius of the fundamental

damental Scheme be 1000, ZP shal be 349, the natural tangent of 19 degrees, 23 minutes, 50 seconds, the half of 38 degrees, 47 minutes, the distance between the North pole and the Zenith in our Latitude of 51 degrees, 53 minutes, or centesmes of a degree. And the South pole being as much under the Horizon as the North is above it, the distance thereof from the Zenith must be the complement of 38 degrees, 47 minutes to a Semicircle, that is, 141 degrees, 53 minutes; and as the half of 38 degrees, 47 minutes, *viz.* 19 degrees, 23 minutes, 50 seconds is the quantity of the angle P E Z, and the tangent thereof the distance from Z to P, so the half of 141 degrees, 53 minutes, *viz.* 70 degrees, 76 minutes, 50 seconds must be the measure of the angle in the circumference between the Zenith and the South, the tangent whereof 2866 must be the distance also, and the tangents of these two arches added together 3215, is the whole diameter of that circle, the half whereof 1607, that is, one Radius, and neer 61 hundred parts of another is the Semidiameter or distance from P to L in the former Scheme, to which extent open the compasses, and set off the distance P L, and therewith draw the circle

WPE

W P E for the six of the clock hour.

The Semidiameters of the other circles are to be found in the same manner; the distance between the Zenith and the Equinoctiall is alwayes equal to the height of the Pole, which in our Latitude is 51 degr. 53 min. and therefore the half thereof 25 degrees, 76 minutes, 50 seconds is the measure of the angle W E B, and the natural tangent thereof 483, which being added to the tangent of the complement 2070, their aggregate 2553 will be the whole diameter of that circle, and 1277 the Radius or Semidiameter by which to draw the Equinoctiall circle E K W.

The Tropique of *Cancer* is 23 degrees, 53 minutes above the Equator, and 66 degrees 47 minutes distant from the Pole, and the Pole in this Latitude is 38 degrees 47 min. distant from the Zenith, which being substracted from 66 degrees 47 minutes, the distance of the Tropique of *Cancer* from the Zenith, will be 28, the half thereof is 14, whose natural tangent 249 being set from Z to T, giveth the point T in the Meridian, by which that parallel must passe; the distance thereof from the Zenith on the North side is E N 90 degrees, and substracting 23 degrees, 53 minutes, the

heighs

Height of the Tropique above the Equator, from 38 degrees, 47 minutes, the height of the Equator above the Horizon, their difference is 14 degrees, 94 minutes, the distance of the Tropique from N under the Horizon; and so the whole distance thereof from Z is 104 degrees, 94 minutes, the half whereof is 52 degrees, 47 minutes, and the natural tangent thereof 1302 added to the former tangent 249, giveth the whole diameter of that circle 1551, whose half 776 is the Semidiameter desired, and gives the center to draw that circle by.

The Tropique of *Capricorn* is 23 degrees, 53 minutes below the Equator, and therefore 113 degrees 53 minutes from the North pole, from which if you deduct, as before, 38 degrees, 47 minutes, the distance of the Pole from the Zenith, the distance of the Tropique of *Capricorn* from the Zenith will be 75 degrees, 6 minutes, and the half thereof 37 degrees, 53 minutes, whose natural tangent 768 being set from Z to F, giveth the point F in the Meridian, by which that parallel must pass: the distance thereof from the Zenith on the North side is Z N 90 degrees, as before; and adding 23 degrees, 53 minutes, the distance of the Tropique from the Equator to 38 degrees,

47 minutes, the distance of the Equator from the Horizon, their aggregate is 62 degrees, the distance of the Tropique from the Horizon, which being added to Z N 90 degrees, their aggregate is 152 degrees, and the half thereof 76 degrees, whose naturall tangent 4011 being added to the former tangent 368, giveth the whole diameter of that circle 4379, whose half 2189 is the Semidiameter desired, and gives the center to draw that circle by.

The distance of the reclining circle NOS from Z to O is 40 degrees, the half thereof 20, whose naturall tangent 364 set from Z to O, giveth the point O in the prime verticall EZW, by which that circle must pass; the distance thereof from the Zenith on the East side is Z E 90 degrees, to which adding 50 degrees, the complement of the former arch, their aggregate 140 degrees is the distance from Z Eastward, and the half thereof 70 degrees, whose natural tangent 2747, being added to the former tangent 364, their aggregate 3111 is the whole diameter of that circle, and the half thereof 1555 is the Semidiameter desired, and gives the center to draw that circle by.

The distance of the declining reclining circle DAG from the Zenith is ZA 35 deg. the

the half thereof 17 degrees, 50 minutes, whose natural tangent 315 being set from Z to A giveth the point by which that circle must passe, and the natural tangent of 72 degr. 50 min. the complement thereof 3171 being added thereto is 3486, the whole diameter of that circle, and the half thereof 1743, the Semidiameter desired, and giveth the center to draw that circle by.

The streight lines C Z A or D Z G are put upon the limbe by help of a line of Chords 30 degrees distant from the Cardinal points N E S W, and must crosse each other at right angles in Z, representing two Azimuths equidistant from the Meridian and prime verticall.

Last of all, the hour circles are thus to be drawn; first, seek the center of the six of clock hour-circle, as formerly directed, making Z E the Radius, and is found at L upon the Meridian line continued from P to L, which cross at right angles in L with the line 8 L 4, extended far enough to serve the turn, make P L the Radius, then shall 8 L 4 be a tangent line thereunto, and the natural tangents of the Equinoctiall hour arches, that is the tangent of 15 degrees 268 for one hour, of 30 degr. 577 for two hours, of 45 degrees 1000 for three hours

of 60 deg. 1732 for four hours, and 75 deg. 3732 for five hours set upon the line from L both wayes; that is, from L to 5 and 7, 4 and 8, and will give the true center of those hour-circles: thus, 5 upon the line 8 L 4 is the center of the hour-circle 5 P 5, and 7 the center of the hour-circle 7 P 7; and so of the rest.

The centers of these hour-circles may be also found upon the line 8 L 4 by the naturall secants of the same Equinoctiall arches, because the hypothenuse in a right angled plain triangle is alwayes the secant of the angle at the base, and the perpendicular the tangent of the same angle: if therefore the tangent set from L doth give the center, the secant set from P shall give that center also. The Scheme with the Lines and circles thereof being thus made plain, we come now to the Art of Dialling it self.

Probl. 3.

Of the severall plains, and to finde their situation.

ALL great Circles of the Sphere, projected upon any plain, howsoever situated, do become streight lines, as any one may experiment upon an ordinary bowle thus. If he saw the Bowle in the midst, and joyne the two parts together again, there will remain upon the circumference of the Bowle, some signe of the former partition, in form of a great Circle of the Sphere: now then, if in any part of that Circle the roundnesse of the bowle be taken off with a smoothing plain or otherwise, as the bowle becomes flat, so will the Circle upon the bowle become a streight line; from whence it follows, that the houre lines of every Diall (being great Circles of the Sphere) drawn upon any plain superficies, must also be streight lines.

Now the art of Dialling consisteth in the artificiall finding out of these lines, and their distances each from other, which do continually varie according to the situation of the plain on which they are projected.

Of these plains there are but three sorts.

1. Parallel to the Horizon, as is the Horizontal only.

2. Perpendicular to the Horizon, as are all erect plains, whether they be such as are direct North, South, East or West, or such as decline from these points of North, South, East, or West.

3. Inclining to the Horizon, or rather Reclining from the Zenith, and these are direct plains reclining and inclining North and South, and reclining and inclining East and West, or Declining-reclining and inclining plains.

To contrive the houre lines upon these severall plains, there are certain Spherical arches and angles, in number six, which must of necessity be known, and divers of these are in some Cases given, in others they are sought.

1. The first is an arch of a great Circle perpendicular to the plain, comprehended betwixt the Zenith and the plain, which is the Reclination, as Z T, Z K, and Z F, in the fundamental Diagram.

2. The second is an arch of the Horizon betwixt the Meridian and Azimuth passing by the poles of the plain, as S V or N C in the Scheme.

3. The third is an arch of the plain betwixt

twixt the Meridian and the Horizon, pre-
scribing the distance of the 12 a clock house
from the horizontal line, as PB in the
Scheme of the 11 th. Probl.

4. The fourth is an arch of the plain be-
twixt the Meridian and the substile, which
limits the distance thereof from the 12 a
clock houre line, as Z R in the Scheme.

5. The fifth is an arch of a great Circle
perpendicular to the plain, comprehended
betwixt the Pole of the World, and the
plain, commonly called the height of the
stile, as P R in the Scheme.

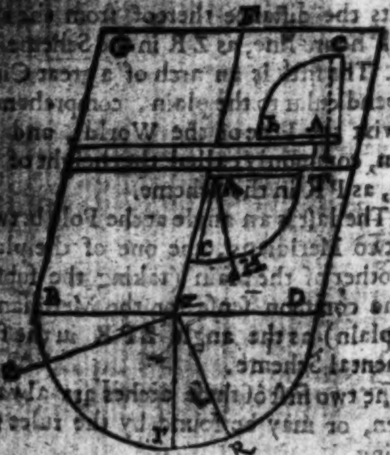
6. The last is an angle at the Pole betwixt
the two Meridians, the one of the place,
the other of the plain (taking the substile
in the common sense for the Meridian of
the plain) as the angle Z P R in the fun-
damental Scheme.

The two first of these arches are alwayes
given, or may be found by the rules fol-
lowing.

To finde the Inclination or Reclination of any plain.

If the plain seeme to be level with the Ho-
rizon, you may try it by laying a ruler
thereupon, and applying the side of your
Quadrant AB to the upper side of the ruler,

so that the center may hang a little over the end of the ruler, and holding up a threed and plummet, so that it may play upon the center, if it shall fall directly upon his level line A C, making no angle therewith, it is an horizontall plain.



If the plain seeme to be verticall, like the wall of an upright building, you may try it by holding the Quadrant so that the threed may fall on the plumb line A C, for then if the side of the Quadrant shall lie close to the plain, it is erect, and a line drawn

drawn by that side of the Quadrant shall be a Verticall line, as the line D E in the figure.

If the plain shall be found to incline to the Horizon, you may finde out the quantity of the inclination after this manner. Apply the side of your Quadrant A C to the plain, so shall the thread upon the limbe give you the inclination required.

Suppose the plain to be B G E D, and the line F Z to be verticall, to which applying the side of your Quadrant A C, the thread upon the limbe shall make the angle G A H the inclination required, whose complement is the declination.

To finde the declination of a plain.

To effect this, there are required two observations, the first is of the horizontal distance of the Sun from the pole of the plain, the second is of the Suns altitude, thereby to get the Azimuth; and these two observations must be made at one instant of time as neer as may be, that the parts of the work may the better agree together.

For the horizontal distance of the Sun from the pole of the plain, apply one edge of the Quadrant to the plain, so that the other may be perpendicular to it, and

the limbe may be towards the Sun; and hold the whole Quadrant horizontal as neer as you can conjecture, then holding a threed and plummet at full liberty, so that the shadow of the threed may passe through the center and limb of the Quadrant, observe then what degrees of the limb the shadow cuts, counting them from that side of the Quadrant which is perpendicular to the horizontall line, those degrees are called the Horizontal distance.

2. At the same instant observe the Suns altitude, by this altitude you may get the Suns Azimuth from the South, by the 10 th. Probleme of the first Chapter hereof.

When you make your observation of the Suns horizontal distance, marke whether the shadow of the threed fall between the South, and the perpendicular side of the Quadrant, or not, for,

1. If the shadow fall between them, then the distance and Azimuth added together do make the declination of the plain, and in this case the declination is upon the same coast whereon the Suns Azimuth is.

2. If the shadow fall not between them, then the difference of the distance and Azimuth is the declination of the plain, and if the Azimuth be the greater of the two, then the

the plain declineth to the same coast where-
on the Azimuth is, but if the distance be the
greater, then the plain declineth to the con-
trary coast to that whereon the Suns Azi-
muth is.

Note here further, that the declination
so found, is alwayes accounted from the
South, and that all declinations are num-
bered from North or South, towards East
or West, and must not exceed 90 deg.

1. If therefore the number of declination
exceed 90, you must take its complement to
180, and the same shall be the plains decli-
nation from the North.

2. If the declination found exceed 180
deg. then the excesse above 180, gives the
plains declination from the North, towards
that Coast which is contrary to the Coast
whereon the Sun is.

By this accounting from North & South,
you may alwayes make your plains decli-
nation not to exceed a Quadrant or 90 de.
And as when it declines nothing, it is a full
South or North plain, so if it decline just
90, it is then a full East or West plain.

These precepts are sufficient to finde the
declination of any plain howsoever situated,
but that there may be no mistake, we will
adde an Example.

1. Ex-

1 *Example.*

Let the horizontall distance from the pole of the plains horizontal line represented in the last diagram by R Z the line of shadow, be 24 degrees, and let the Suns Azimuth from the South be 40 deg. describe the circle B C M P, the which shall represent the horizontal circle, and draw the diameter B A C representing the horizontal

line of the plain, and the diameter M P representing the poles of the plains horizontal line; then by a line of Chords set off your horizontal distance 24 degrees (found by observation in the afternoon) from P to G, and

G, and from G to S set off the suns Azimuth 40 degrees, so shall the point S represent the South, N the North, E the East, and W the West.

Now because the line of shadow A G, falleth between P the pole of the plains horizontal line, and S the South point; therefore according to the former direction, I adde the horizontal distance P G 24 deg. to the Suns Azimuth G S 40 deg, and their aggregate is P S 64 deg. the declination sought; and in this case it is upon the same coast with the sun, that is West, according to the rule given, and as the figure it selfe sheweth, the East and North points being hid from our sight by the plain it selfe; this therefore is a South plain declining West 64 degrees.

2. *Example.*

Let the horizontal distance taken in the afternoon by observation, be 67 degrees, and the Suns Azimuth from the South 42 deg. be given, then draw, as before, the Circle B C M P, and from P to H set off the horizontal distance 67 deg. from H to S the suns Azimuth, 42 deg. Now then, because the South point doth fall between P the pole of the plains horizontal line, and H,

G, and from G to S set off the suns Azimuth 40 degrees, so shall the point S represent the South, N the North, E the East, and W the West.

Now because the line of shadow A G falleth between the pole of the plains horizontal line, and the South point, therefore according to the former direction I adde the horizontal distance P G 24 deg. to the Suns Azimuth G S 40 deg. the aggregate is P S 64 deg. the declination sought, and in this case it is upon the same coast with the sun, that is West, according to the rule given, and as the figure it selfe sheweth, the East and North points being

the horizontal distance, I deduct the Suns Azimuth H S 43 degrees, from H P the horizontall distance, and their difference is S P 25 degrees, the declination sought; and because the horizontal distance is greater than the Azimuth, therefore the declination is contrary to the Coast of the Sun. This then is a South plain declining East 25 degrees.

To finde a Meridian line upon an Horizontal plain.

If your plain be levell with your Horizon, draw thereon the Circle B C M P, then holding a threed and plummet, so as the shadow

shadow thereof may fall upon the center, and draw in the last diagram the line of shadow H A: then if the Suns Azimuth shall be 50 deg. and the line of shadow taken in the afternoon, set off the 50 deg. from H to S, and the line S N shall be the Meridian line desired.

Probl. 4.

To draw the houre lines upon the Horizontal plain.

This plane in respect of the Poles thereof, which lie in the Vertex and Nadir of the place may be called vertical, in respect of the plane it self, which is parallel to the Horizon, horizontal, howsoever it be termed, the making of the Dial is the same, and there is but one onely arch of the Meridian betwixt the pole of the world and the plane required to the artificiall projecting of the hour-lines thereof, which being the height of the pole above the horizon (equal to the height of the stile above the plane) is alwayes given, by the help whereof we may presently proceed to calculate the hour-distances in manner following.

This plane is represented in the fundamental

mental Diagram by the outward circle ESWN, in which the diameter S N drawn from the South to the North may go both for the Meridian line, and the Meridian circle, Z for the Zenith, P for the pole of the world, and the circles drawn through P for the hour-circles of 1, 2, 3, 4, &c. as they are numbred from the Meridian, and limit the distance of each hour line from the Meridian upon the plane, according to the arches of the Horizon, N 11, N 10, N 9, &c. which by the severall Triangles S P 11, S P 10, S P 9, or their verticals N P 11, N P 10, N P 9 may thus be found; because every quarter of the Horizon is alike, you may begin with which you will, and resolve each hours distance, either by the small Triangle N P 11, or the verticall Triangle K P 11, In the Triangle P N 11, the side P N is alwayes given, and is the height of the pole above the horizon, the which at *London* is 51 deg. 53 min, and the angle at P is given one hours distance from the Meridian, whose measure in the Equinoctiall is 15 deg. & the angle at N is alwayes right, that is 90 deg. wherefore by the first case of right angled spherical Triangles, the perpendicular N 11 may thus be found.

As

As Radius 90, 10.000000
To the tangent of NP 11, 15 d. 9.428052
So is the sine of PN 51.53. 9.893725

To the tangent of N 11, 22.85. 9.321777

Which is the distance of the hours of 1 and 11, on each side of the Meridian, thus in all respects must you finde the distance of 2 and 10 of clock, by resolving the triangle N P 10, and of 3 and 9 of clock, by resolving the triangle N P 9; and so of the rest; in which, as the angle at P increaseth which for 2 hours is 30 degrees, for 3 hours 45 degr. for 4 hours 60 degr. for 5 hours 75 degr. so will the arches of the Horizon N 10, N 9, N 8, N 7, vary proportionably, and give each hours true distance from the Meridian, which is the thing desired.

Probl. 5.

To draw the hour-lines upon a direct South or North plane.

Every perpendicular plane, whether direct or declining, lieth in some Azimuth or other; as here the South wall or plane doth lie in the prime vertical or Azimuth of East and West, represented in the

the fundamental Diagram by the line E Z W, and therefore it cutteth the Meridian of the place at right angles in the Zenith, and hath the two poles of the plane seated in the North and South intersections of the Meridian and Horizon; and because the plane hideth the North pole from our sight, we may therefore conclude, (it being a general rule that every plane hath that pole depressed, or raised above it, which lieth open unto it) that the South pole is elevated thereupon, and the stile of this Diall must look downwards thereunto, erected above the plane the height of the Antartick Pole, which being an arch of the Meridian betwixt the South pole and the Nadir, is equall to the opposite part thereof, betwixt the North pole and the Zenith; and therefore the complement of the North pole above the horizon.

Suppose then that P in the fundamental Scheme, be now the South pole, and N the South part of the Meridian, S the North; then do all the hour-circles from the pole cut the line E Z W, representing the plane unequally, as the hour-lines will do upon the plane it self, and as it doth appear by the figures set at the end of every hour line in the Scheme. Now having already the

poles

poles elevation given, as was in the horizontal, there is nothing else to be done, but to calculate the true hour-distances upon the line E Z W from the meridian S Z N; and then to proceed, as formerly, and note that because the houres equidistant on both sides the meridian, are equal upon the plane, the one half being found, the other is also had, you may therefore begin with which side you will.

In the triangle Z P 11, right angled at Z, I have Z P given, the complement of the height of the pole 38 deg. 47 min. the which is also the height of the stile to this Diall, and the angle at P 15 degrees one hours distance from the meridian upon the Equator to finde the side Z 11, for which by the first case of right angled sphericall triangles, the proportion is, as before.

As the Radius 90,		10.000000
To the sine of P Z 38.47.		9.793263
So is the tangent of ZP11,	15d.	9.428052
To the tangent of Z 11,	9.47.	9.221315

And thus in all respects must you finde the distance of 2 and 10, of 3 and 9; and so forward, as was directed for the houres in the horizontal plane.

The

The North plane is but the back side of the South, lying in the same Azimuth with it, & represented in the Scheme by the back part of the same streight line E Z W, whatsoever therefore is said of the South plane may be applied to the North; because as the South pole is above the South plane 38 degr. 47 min, so is the North pole under the North plane as much, and each stile must respect his own pole, onely the meridian upon this plane representeth the midnight, and not the noon, and the hours about it 9, 10, 11, and 1, 2, 3, are altogether uselesse, because the Sun in his greatest northern declination hath but 39 degr. 90 min. of amplitude in this our Latitude; and therefore riseth but 22 min. before 4 in the morning, and setteth so much after 8 at night; neither can it shine upon this plane longer then 35 min. past 7 in the morning, and returning to it as much before 5 at night, because then the Sun passeth on the North side of the prime vertical, in which this plane lieth, and cometh upon the South.

Now therefore to make this Dial, is but to turn the South Dial upside down, and leave out all the superfluous hours between 5 and 7, 4 and 8, and the Diall to the

North plane is made to your hand.

The Geometricall projection.

To project these and the Horizontall Dials, do thus: First, draw the perpendicular line C E B, which is the twelve of clock hour, crosse it at right angles with 6 C 6, which is the six of clock hour; then take with your compasses 60 deg. from a line of Chords, and making C the center draw the circle 6 E 6, representing the azimuth in which the plane doth lie; this done, take from the same Chord all the hour distances, and setting one foot of your compasses

in E, with the other mark out those hour distances before found by calculation, both wayes

wayes upon the circle 6 E 6; streight lines drawn from the center C to those pricks in the circle are the true hour-lines desired.

Having drawn all the hour-lines, take from the same line of Chords the arch of your poles elevation, or stile above the plane, and place it from E to O, draw the right line C O A representing the axis or height of the stile, from any part of the meridian draw a line parallel to 6 C 6, as is BA, so it shall make a triangle, the fittest form to support the stile at the true height; let the line 6 C 6 be horizontal, the triangular stile C B A erected at right angles over the 12 of clock line, and then is the Diall perfected either for the Horizontal, or the direct North and South planes.

Probl. 6.

To draw the hour-lines upon the direct East or West planes.

As the planes of South and North Dials do lie in the Azimuth of East and West, and their poles in the South and North parts of the meridian; so do the planes of East and West Dials lie in the South and North azimuth, and their poles in the East and West part of the Horizon,

rizon,

rizon, from whence these Dials receive their denomination, and because they are parallel to the meridian line in the fundamental Scheme S Z N, some call them meridian planes.

And because the meridian, in which this plane lieth, is one of the hour-circles, and no plane that lieth in any of the hour-circles can cut the axis of the world, but must be parallel thereunto; therefore the hour lines of all such planes are also parallel each to other, and in the fundamental Scheme may be represented in this manner.

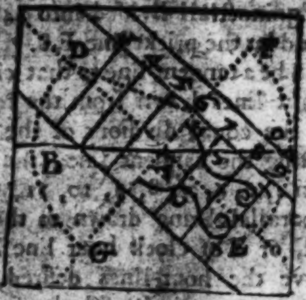
Let N E S W in this case be supposed to be the Equinoctiall divided into 24 equall parts, and let the prickt line E. 8. 7. parallel to Z S be a tangent line to that circle in E, straight lines drawn from the center Z thorow the equal divisions of the limbe, intersecting the tangent line, shall give points in 4, 5, 6, 7, 8, 9, 10, 11, thorow which parallels being drawn to the prime vertical, or 6 of clock hour line E Z W, you have the hour-lines desired, which may for more certainties sake be found by tangents also; for making Z E of the former Scheme to be the Radius, and E 8. 7. a tangent line, as before; then shall the na-

tural

small tangent of 15 degr. (so taken from a diagonall scale equall to the Radius, and set both wayes from E upon the tangent line E 8, 4.) gives the distance of the houres of 5 and 7, the tangent of 30 degr. the distance of the hours of 4 and 8, and the tangent of 45 degr. the distance of the hours of 3 and 9, &c. from the six of clock hour, as before; and is a generall rule for all Latitudes whatsoever.

The Geometricall projection.

Proceed then to make the Diall, and first draw the horizontal line B A upon any part



thereof, as at A, draw two obscure arches D B G and F C E, and with that line of Chords, with which the arches were drawn

set

set off 38 deg. 47 min. the height of the Equator at *London* from B to D, and from C to E set off likewise 51 deg. 53 min. the height of the Pole from B to G, and from C to F, and draw the streight line D A E, representing the Equinoctial, as is manifest by the angle B A D 38 deg. 47 min. which the Horizon makes with the Equator; and the streight line F A G representing the Axis of the World, as is manifest by the angle F A C 51 deg. 53 min. which the Pole and Horizon make, and this wil be also the six of clock houre, or substile of this Diall, seeing the plain it selfe lieth in the Meridian, 90 deg. distant. And because the top of the Stile (which may be a streight pin fixed in the point A) doth give the shadow in all plains that are parallel to the Axis, it will be neceesary to proportion the stile to the plain, that the hour lines may be enlarged or contracted according to the length thereof, the which is done in this manner. Let the length of the plain from A be given in some known parts, then because the extream houre of the East Dial is 11, in the West 1, reckoning 15 deg. to every houre from six, the arch of the Equator will be 75 deg. and therefore in the right angled plain triangle A H E, we have given the base A E, which

is the length of the plain from A, and the angle A H E 75 deg. to finde the perpendicular H A, for which (by the 1 Case of right angled plain Triangles) the proportion wil be,

As the Radius, 90	10.000000
To the Base A E, 3.48	2.541579
So the Tangent of A E H, 15	9.428052
To the perpendicular H A, 93	1.969631

The length of the stile being thus proportioned to the plain, make that the Radius of a Circle, and then the Equator D A E shall be a Tangent line thereunto, and therefore, the naturall Tangent of 15 deg. being set upon the Equinoctiall D A E both wayes from A, shall give the points of 5 and 7: the Tangent of 30 deg. the points of 8 and 4, &c. through which streight lines being drawn parallel to the six a clock houre, you have at one work made both the East and West Dials, only remember that because the Sun riseth before 4 in *Cancer*, and setteth after 8, you must adde two houres before six in the East Diall, and two houres after six in the West, that so the plain may have as many houres as it is capable of.

The West Dial is the same in all respects with

with the East, only the arch B D, or the height of the Equator, must be drawn on the right hand of the center A for the West Dial, and on the left for the East, that so the houre lines crossing it at right angles, may respect the Poles of the world to which they are parallel.

Probl. 7.

To draw the houre-lines upon a South or North erect plain declining East or West, to any declination given.

Every erect plain lieth under some Azimuth or other, and those only are said to decline which differ from the Meridian and Prime Vertical. The declination therefore being attained by the rules already given, (or by what other means you like best) we come to the calculation of the Diall it selfe, represented in the fundamentall Scheme by the right line G Z D, the Poles whereof are C and V, the declination from the South Easterly N C, or North Westerly S V, 25 deg. supposing now S to be North, and N South; W East, and E the West point, the houre circles proper to this plain are the black lines pas-

sing through the Pole P, and crossing upon the plain G Z D, wherein note generally that where they run neerest together, thereabouts must the sub-stile stand, and alwayes on the contrary side to the declination, as in this example declining East, the stile must stand on the West side (supposing P to be the South Pole) between Z and D, the reason whereof doth manifestly appear; because the Sun rising East, sendeth the shadow of the Axis West, and alwayes to the opposite part of the Meridian wherein he is, wherefore reason enforceth, that the morning houres be put on the West side of the Meridian, as the evening houres are on the East, and from the same ground that the substile of every plain representing the Meridian thereof, must alwayes stand on the contrary side to the declination of the plain and that the houre-lines must there run neerest together, because the Sun in that position is at right angles with the plain. For the making of this Diall three things must be found.

1. The elevation of the Pole above the plain, represented by P R, which is the height of the stile, and is an arch of the Meridian of the plain, between the Pole of the world and the plain.

2. The

2. The distance of the substile from the Meridian, represented by Z R, and is an arch of the plain between the Meridian and the substile.

3. The angle Z P R, which is an arch between the substile PR the meridian of the plain, and the line P Z the meridian of the place, and these are thus found.

Because the substile is the Meridian of the plain, it must be part of a great circle passing through the pole of the world, and crossing the plain at right angles, therefore in the supposed right angled triangle PRZ, (for yet the place of R is not found) you have given the base P Z 38 deg. 47 min. and the angle P Z R the complement of the declination 65 deg. and the supposed right angle at R, to finde the side P R, which is the height of the stile as aforesaid, but as yet the place unknown: wherefore by the 8 Case of right angled Spherical Triangles the analogie is,

As the Radius,	10.00 00
To the sine of P Z, 38.47	9.795863
So the sine of P Z R, 65	9.957275
To the sine of P R, 34.32	9.753138

Secondly, you may finde ZR the distance of the substile from the meridian, by the 7 case of

of right angled Spherical Triangles.

As the Radius, 90	10.000000
To the Co-sine of P Z R, 65	9.629378
So is the tangent of P Z, 38.47	9.900138
To the tangent of Z R, 18.70	9.529516

These things given, the angle at P between the two meridians may be found by the 9 Case of right angled Sphericall Triangles, for the proportion is,

As the Radius, 90	10.000000
To the Co-sine of P Z, 38.47	9.893725
So the Tangent of PZR, 65	10.331327
To the Co-tang. of RPZ, 30.78	10,225052

Having thus found the angle between the Meridians to be 30 deg. 78 min. you may conclude from thence, that the substile shall fall between the 2d. & third houres distance from the Meridian of the place, and therefore between 9 and 10 of the clock in the morning, because the plain declineth East from us, 9 of the clock being 45 deg. from the Meridian, and 10 of the clock 30 deg. distant, now therefore let fall a perpendicular between 9 and 10, the better to inform the fancie in the rest of the work, and this shall

ſhall make up the Triangle P R Z before mentioned and ſuppoſed, which being found we may calculate all the houre diſtances by the firſt caſe of right angled ſphericall Triangles. For,

As the Radius,
Is to the ſine of the baſe P R;
So is the Tangent of the angle at the perpendicular, R P 9,
To the tangent of R 9 the perpendicular.

The angle at P is alwayes the Equinoctiall diſtance of the houre line from the ſubſtile, and may thus be found: If the angle between the Meridians be leſſe than the houre diſtance, ſubſtract the diſtance of the ſubſtile from the houre diſtance; if greater ſubſtract the houre diſtance from that, and their difference ſhall give you the Equinoctiall diſtance required.

Thus in our Example, the angle between the Meridians was found to be 30 deg. 78 m. and the diſtance of 9 of the clock from 12 is three houres, or 45 deg. if therefore I ſubſtract 30 deg. 78 min. from 45 deg. the remainder will be 14 deg. 22 min. the diſtance of 9 of the clock from the ſubſtile. Again, the diſtance of 10 of the clock from the Meridian is 30 deg. and therefore

if I ſubſtract 30 deg. from 30 deg. 78 min. the diſtance of 10 of the clock from the ſubſtile will be 78 centeſims or parts of a degree: the reſt of the houres and parts are eaſily found by a continual addition of 15 deg. for every houre, 7 deg. 50 min. for half an houre, 3 deg. 75 min. for a quarter of an houre, as in the Table following you may perceive, the which conſiſts of three columns, the firſt containeth the houres, the ſecond their Equinoctiall diſtances from the ſubſtile, the third and laſt the houre arches, computed by the former proportion in this manner.

As the Radius, 90	10.000000
Is to the ſine of P R, 34.32	9.751136
So is the tang. of R P 9, 14.22	9.403824
To the tangent of R 9, 8.13	9.154960

H	*Æqu.*	*Arches*	H	*Æqu.*	*Arches*
4	89 22	88 61	11	15 78	9 05
5	74 22	63 38	12	30 78	18 56
6	59 22	43 43	1	45 78	30 08
7	44 22	28 75	2	60 78	45 23
8	29 22	17 50	3	75 78	65 80
9	14 22	8 13	4	90 78	88 61
	merid.	*ſubſtil*			
10	00 78	00 44			

The

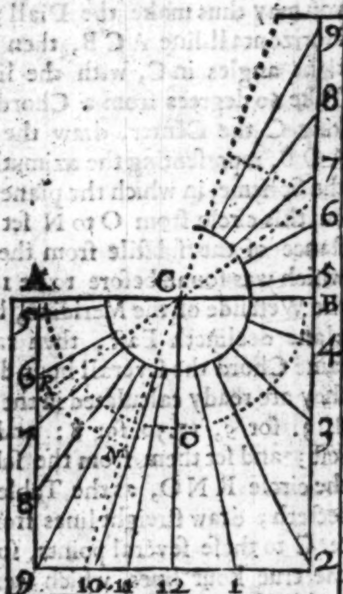
The Geometricall Projection.

Having calculated the hour distances, you may thus make the Diall; Draw the Horizontall line A C B, then crosse it at right angles in C, with the line C O 12. Take 60 degrees from a Chord, and making C the Center, draw the Semicircle A O B, representing the azimuth G Z D in the Scheme, in which the plane lieth; upon this circle from O to N set off the distance of the substile from the Meridian, which was found before to be 18.70. upon the West side of the Meridian, because this plane declineth East, then take off the same Chord the severall hour-distances, as they are ready calculated in the table, *viz.* 8.13. for 9, 17.50. for 8: and so of the rest; and set them from the substile upon the circle R N O, as the Table it self directeth; draw streight lines from the center C to these several points, so have you the true hour lines, which were desired: and lastly, take from the same Chord the heighth of the stile found to be 34. 32. which being set from N to R, and a streight line drawn from C through R representing the axis, the Diall is finished for use.

In applying it to any wall or plane, let

The Occidentall Diall.

Having calculated the hour distances you may thus make the Diall; Draw the horizontall line A C B, then crosse it at right angles in C, with the line C O ra[illegible] of 60 degrees upon a Chord, and ma[illegible] C the Center, draw the Semicircle [illegible] declinatie azimuth G N D in [illegible] in which the plane lieth; up[illegible] from O to N set off the di[illegible] of the Stile from the Meridian, [illegible] before to be 18. 50. upon [illegible] Verticale [illegible] because [illegible] then take off the [illegible] distances, as [illegible] table, &c. [illegible] and so of the [illegible] and set them from the Substile upon [illegible] N O, as the Table it self di[illegible] draw streight lines from the cen[illegible] to those severall points, so have you [illegible] defined: [illegible] take from the same Chord the height of the Stile found to be 34. 53. which being set from N to R, and a streight line drawn from C through R representing the axis, the Diall is finished for use.



In applying it to any wall or plane, let

ACB be horizontal, CO perpendicular, and the side or axis of the stile, CR pointing downwards, erected over the substile line CN; so have you fitted a Diall for any South plane declining East 25 degrees.

Nay, thus have you made four Dials in one, *viz.* a South declining East and West 25 degrees, and a North declining East or West as much; to make this plainly appear, suppose in the fundamental Scheme if N were again the North part of the horizon, P the North pole, and that GZD were a North declining West 25 degrees, then do all the hour-circles crosse the same plane, as they did the former; onely DZ which was in the former the East side will now be the West: and consequently the afternoon hours must stand where the forenoon hours did; the stile also, which in the East declining stood between 9 and 10, must now stand between 2 and 3 of the afternoon hours. And lest there should be yet any doubt conceived, I have drawn to the South declining East 25, the North declining West as much; from which to make the South declining West, and North declining East, you need to do no more then prick these hour lines through the paper, and draw them again on the other

side,

side, stile and all; so shall they serve the turn, if you place the morning hours in the one, where the afternoon were in the other.

APPENDIX.

To draw the hour lines upon any plane declining far East or west, without respect to the Center.

THe ordinary way is with a Beam-compasse of 16, 18, or 20 foot long, to draw the Diall upon a large floor, and then to cut off the hours, stile and all, at 10, 12, or 14 foot distance from the center, but this being too mechanical for them that have any Trigonometrical skill, I omit, and rather commend the way following; by help whereof you may upon half a sheet of paper make a perfect model of your Diall, to what largenesse you please, without any regard at all to the Center.

Suppose the wall or plane D Z G, on which you would make a Diall to decline from N to C, that is from the South Easterly 82 degrees, 62 min, set down the *Data*, and

and by them seek the *Quæsita*, according to the former directions.

The *Data* or things given are two.

1. PS the poles elevation 51 degrees, 53 minutes.

2. SA, the planes declination southeast 83 deg. 62 min.

The *Quæsita* or things sought are three.

1. PR the height of the stile 3 degrees 97 minutes.

2. ZR, the distance of the substile from the meridian 38 deg. 30 min.

3. ZPR, the angle of the meridian of the plane with the meridian of the place 85 degrees, which being found, according to the former directions, the substile line must fall within five degrees of six of the clock, because 85 degrees wanteth but 5 of 90, the distance of 6 from 12. Now therefore make a table, according to this example following, wherein set down the houres from 12, as they are equidistant from the meridian, and unto them adjoyn their Equinoctial distances, and write Meridian and substile between the hours of 6 and 7, and write 5 degrees against the hour of 6, 10 degrees against the hour of 7, and to the Equinoctial distances of each hour adde the natural tangents of those distan-

ces,

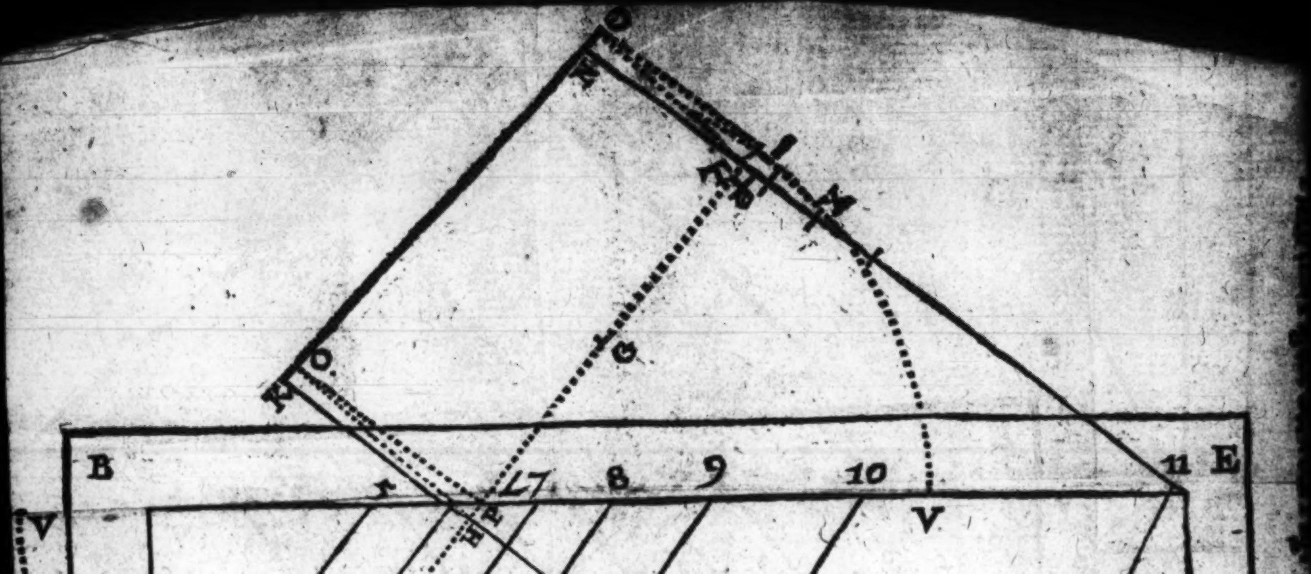
tes, as here you see. So is the Table prepared for use, by which you may easily frame the Diall to what greatnesse you will, after this manner.

Hours		Equ. dist.		Tang.
4	8	35	0	700
5	7	20	0	364
6	6	5	0	087
		Meridiā		*Substile*
7	5	10	0	176
8	4	25	0	466
9	3	40	0	839
10	2	55	0	1.428
11	1	70	0	2.747
12	12	85	0	11.430

The Geometricall projection.

Proportion the plane B C D E, whereon you will draw the Diall to what scantling you think fit. Let V P represent the horizontall line, upon any part thereof, as at P, make choice of a fit place for the perpendicular stile (though afterwards you may use another forme) neer about the upper part of the plane, because the great angle betwen the two Meridians maketh the substile, which must passe thorow the point

P, to



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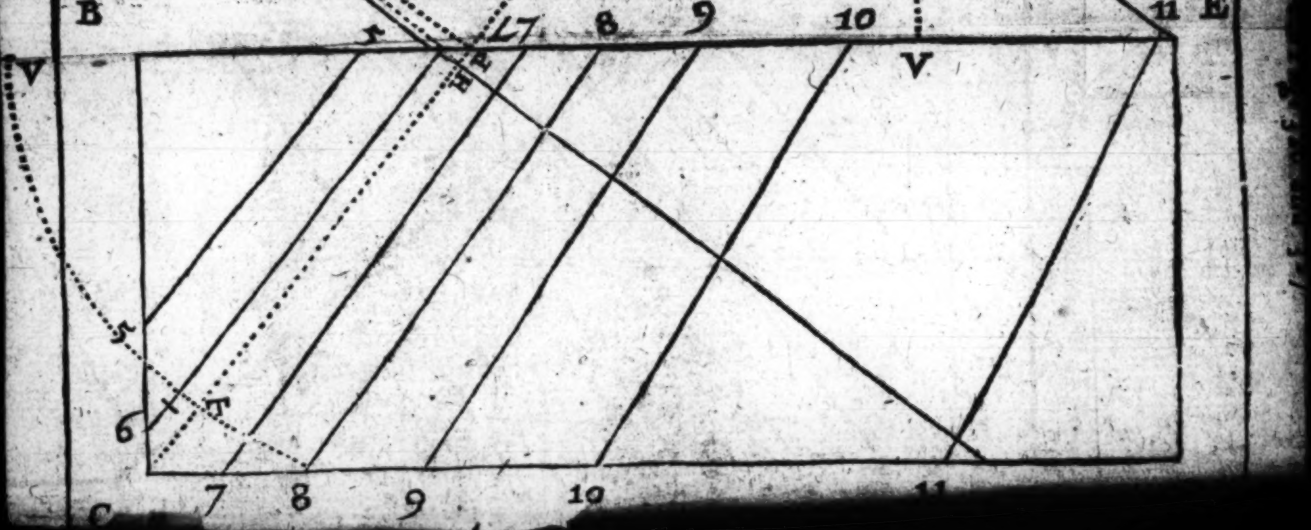
8
9
10
11
E
V
V


B
E
V
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5
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H
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T
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C

P, to fall so near the 6 of clock hour, as that there may be but one hour placed above it, if you desire to have the hour of 11 upon the plane, which is more useful then 4, let P be the center, and with any Chord (the greater the better) make two obscure arches; one above the horizontal line, the other under it, and with the same Chord set off the arch of 51. 70. which is the angle between the substile and horizon, and is the complement of the angle between the substile and meridian, and set it from V to T both wayes, then draw the streight line T P T, which shall be the substile of [illegible] Diall.

This done, proportion the length of P O the [illegible]pendicular stile to what scantling [illegible]ill, and from a diagonal Scale fitted to the Radius of your intended perpendicular stile, set off 69, the natural tangent of 3 degrees 97 min. the height of the stile found by calculation from P to H. Then by a scale proportional to the Radius P O, and at the point H draw the Equinoctial K H 11, cutting the substile at right angles; which if rightly drawn, will cut the horizontal line at 6 of the clock, and make an angle of 38 deg. 30 min. with the horizon, equal to the distance of the substile from the Meridian,

ridian, upon this Equinoctial line making H O the Radius, set off 364, the natural tangent of 20 degrees from H upwards for the 5 of clock hour, and 2747 the natural tangent of 70 degrees, from H downwards for the 11 of clock hour, if these two hour distances fit not the plane to your liking, make P O greater or lesser, as you see cause, for according to this, the distance of H from P, (by which the Equinoctial line must be drawn) the length of H O, and the width of all the hour lines must vary proportionably, but if they fit the plane, then by your scale proportioned to the Radius H O, and the help of the natural tangents set the hours upon the Equinoctial, after this manner: In the right angled plain triangle H G 11, having the perpendicular H G equal to H O given in some known parts: as suppose 206, that is 2 inches and 6 parts of an inch, and the angle H G 11, 70 degrees, the base H 11 may be found by the first case of right angled plain triangles: for,

As the Radius 90	10.000000
Is to the perpendicular HG 206,	2.313867
So is the tangent of HG11, 70.	10.438935
To the base H11, 566.	2.752802

Which

Which is 5 inches, and 66 hundred parts for the diſtance of 11 a clock from the point H, and will be the ſame with thoſe points ſet off by the natural tangents in the Table. Having done with this Equinoctiall, you muſt do the like with another: to finde the place whereof, it will be neceſſary firſt to know the length of the whole line from H the Equinoctial to the center of the Diall in parts of the perpendicular ſtile P O, if you will work by the ſcale of inches, or elſe the length in natural tangents, if you will uſe a diagonall Scale: firſt therefore, to finde the length thereof in inch-meaſure, we have given in the right angled plain triangle H O P, the baſe OP, and the angle at O to finde H P, and in the triangle O P center. We have given the perpendicular O P, and the angle P O center the complement of the former, to finde H center: wherefore, by the firſt caſe of right angled plain triangles:

As the Radius 90	10.000000
Is to the baſe O P 206;	2.313867
So is the tang. of HOP 3.87.	8.841364
To the perpendicular P H 14.	1.155231

Again,

Again,

As the Radius 90,	10.000000
Is to the perpend. OP 206,	2.313867
So is the tang. P O center 86.3.	11.158636
To the base P center 2972	3.472403

Adde the two lines of 014 and 2972 together, and you have the whole line H center 2986 in parts of the Radius P O, *viz.* 29 inches, and 86 parts; out of this line abate what parts you will, suppose 343, that is, 3 inches and 43 parts, and then the remainer will be 2643. Now if you set 343 from H to I, the triangle I O center will be equiangled with the former, and I center being given, to finde L O, the proportion is;

As H center the first base 2986, *co.ar.*	6.524911
Is to HO, the first perpend. 206.	2.313867
So is I center the 2d. base 2643,	3.422097
To I O the 2d. perpend. 182,	2.260875

Having thus found the length of I O to be one inch, and 82 parts; make that the Radius, and then N T 4 shall be a tangent line thereunto, upon which, according to this new Radius, set off the hour-distances before

before found, and so have you 2 pricks, by which you may draw the height of the stile O O, and the hour-lines for the Dial.

The length of H center in natural tangents, is thus found, H P 069 is the tangent line of the angle H O P 3 deg. 97 min. and by the same reason P center 14421 is the tangent line of P O center 86.3. the complement of the other, and therefore these two tangents added together do make 14490, the length of the substile H center, that is, 14 times the Radius, and 49 parts, out of which substract what number of parts you will, the rest is the distance from the second Equinoctial to the center in natural tangents; suppose 158 to be substracted, that is, one radius, and 58 parts, which set from H to T, in proportion to the Radius H O, and from the point T draw the line N T 4 parallel to the former Equinoctial, and there will remain from T to the center 1291. Now to finde the length of L O, the proportion, by the *16th.* of the second, will be

As H center 1449,	co. ar.	6.838932
Is to H O 321,		2.506505
So is T center 1291,		3.110926
To T O 286,		2.456363

Now then if you set 286 from T to O in the same measure, from which you took H O, then may you draw O N O, and the tangents in the Table set upon the line NT in proportion to this new radius T O, you shall have two pricks, by which to draw the hour-lines, as before.

Probl. 8.

To draw the hour lines upon any direct plane, reclining or inclining East or West.

Hitherto we have only spoken of such planes, as are either parallel or perpendicular to the horizon, all which except the horizontal, lie in the plane of some azimuth or other. The rest that follow are reclining or inclining planes, according to the respect of the upper or nether faces of the planes, in those that recline, the base is a line in the plane, parallel to the Horizon or Meridian, and alwayes scituate in some azimuth or other: thus

thus the base of the East and West recli-
ning planes lie in the Meridian, or South and North azimuth, and the poles thereof in the prime vertical, but the plane it self in some circle of position (as it is Astrologically taken) which is a great circle of the Sphere, passing by the North or South intersections of the meridian and horizon, and falling East or West from the Zenith upon the prime vertical, as much as the poles of the plane are elevated and depressed above and under the horizon. And this kinde of plane rightly conceived and represented in the fundamental Scheme by N O S, is no other but an erect declining plane in any Countrey, where the pole is elevated the complement of ours: for if you consider the Sphere, it is apparent, that as all the azimuths, representing the decliners, do crosse the prime vertical in the Zenith, and fall at right angles upon the horizon, so do all the circles of position, representing the reclining and inclining East or West planes crosse the horizon in the North and South points of the Meridian, and fall at right angles upon the prime vertical. From which analogie it commeth to passe, that making a Diall declining 30 degr. from the Meridian, it shall be the

same

ſame that a reclining 30 degr. from the Zenith, and contrary, onely changing the poles elevation into the complement thereof, because the prime vertical in this caſe is ſuppoſed to be the horizon, above which the pole is alwayes elevated the complement of the height thereof above the horizon.

And therefore the poles elevation and the planes reclination being given, which for the one ſuppoſe to be, as before, 51 deg. 53 min. and the other, that is, the reclination 35 degrees towards the Weſt, we muſt finde (as in all decliners) firſt the height of the pole above the plane, which in the fundamental diagram is P R, part of the meridian of the plane between the Pole of the world and the plane. 2. The diſtance thereof from the meridian of the place, which is N R part of the plane betwixt the ſubſtile and the meridian. 3. The angle betwixt the two meridians N P R, by which you may calculate the hour diſtances, as in the decliners.

Firſt, therefore in the ſuppoſed triangle N P R (becauſe you know not yet where R ſhall fall) you have the right angle at R the ſide oppoſite P N 51 degr. 53 min. and the angle at N, whoſe meaſure is the reclination

nation Z O 35 degr. to finde the side PR, the height of the stile, or poles elevation above the plane, wherefore, by the eighth case of right angled spherical triangles, the analogie is

As the Radius 90,	10.000000
Is to the sine of P N 51.53.	9.893725
So is the sine P N R 35.	9.758591
To the sine of the side PR 26.69.	9.652316

Secondly, you may finde the side N R, which is the distance of the substile from the meridian, by the seventh case of right angled spherical triangles; for

As the Radius 90,	10.000000
Is to the cosine of PNR 35.	9.913364
So is the tangent of P N 51.53.	10.099861
To the tangent of N R 45.87.	10.013225

Thirdly, the angle at P between the two meridians may be found by the ninth case of right angled spherical triangles.

As the Radius 90,	10.000000
Is to the co-sine of P N, 51.53.	9.793864
So is the tangent of P N R 35.	9.845227
To the co-tangent of RPN 66.46.	9.639091

The

The angle at P being 66 deg. 46 min. the perpendicular P R must needs fall somewhat neer the middle between 7 and 8 of the clock; if then you deduct the Equinoctial distance of 8, which is 60, from 66 deg. 46 min. the Equinoctial distance of 8 of the clock from the substile will be 6 deg. 46 min. again, if you deduct 66 degr. 47 min. from 75 deg. the distance of 7 from the Meridian, the Equinoctiall distance of 7 from the substile will be 08. deg. 53 min. the rest are found by the continual addition of 15 deg. for an hour: thus, 15 degr. and 6 degr. 47 min. do make 21 deg. 47 min. for 9 of the clock; and so of the rest. And now the hour distances upon the plane may be found by the first case of right angled spherical triangles; for

As the Radius 90,	10.000000
Is to the sine of P R 26.69.	9.652404
So is the tangent of R P 8, 6.46.	9.053956
To the tangent of R 8, 2.91.	8.706360

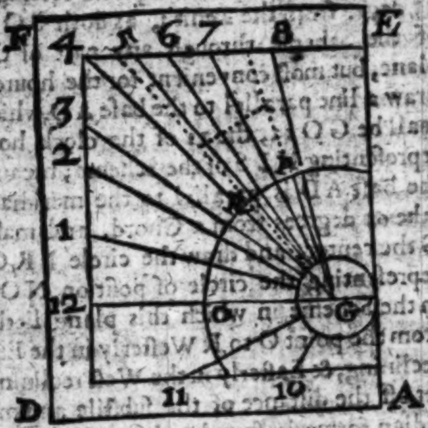
These 2 deg. 91 min. are the true distance of 8 of the clock from the substile. And there is no other difference at all in calculating the rest of the hours, but increasing the angle at P, acccording to each hours Equi-

Equinoctial distance from the substile.

The Geometrical Projection.

Having calculated the hour distances, you shall thus make the Diall; let A D be the base or horizontall line of the plane parallel to N Z S, the meridian line of the Scheme. And A D E F the plane reclining 35 degr. from the Zenith, as doth S O N of the Scheme through any part of the plane, but most convenient for the houres, draw a line parallel to the base A D, which shall be G O 12, the 12 of the clock hour representing N Z S of the Scheme; because the base A D is parallel to the meridian, take 60 degrees from a Chord, and make G the center, and draw the circle P R O, representing the circle of position N O S in the Scheme in which this plane lieth; from the point O to R Westerly in the East reclining, & Easterly in the West reclining, set off the distance of the substile and meridian formerly found to be 45 degrees, 87 min. And draw the thick line G R for the substile, agreeable to P R in the Scheme, G O in the Diall representing the arch P N, and O R in the plane the arch N R in the Scheme. From the point R of the substile both wayes set off the hour distan-

ces, by help of the Chord, for 8 of the clock 2 degr. 91 min. and so of the rest; and draw streight lines from the center G through those points, which shall be the true hour lines desired. Last of all, the height of the stile P R 26 degr. 69 min. being set from R to P, draw the streight line



G P for the axis of the stile, which must give the shadow on the diall, Erect G P at the angle R G P perpendicularly over the substile line G R, and let the point P be directed to the North pole, G O 12 placed in the Meridian, the center G representing the South,

South, and the plane at E F elevated above the horizon 55 degrees; so have you finished this diall for use, onely remember, because the Sun riseth but a little before 4, and setteth a little after 8, to leave out the hours of 3 and 9, and put on all the rest.

And thus you have the projection of four Dials in one; for that which is the West recliner is also the East incliner, if you take the complements of the recliners hours unto 12, and that but from 3 in the afternoon till 8 at night: again, if you draw the same lines on the other side of your paper, and change the houres of 8, 7, 6, &c. into 4, 5, 6, &c. you have the East recliner, and the complement of the East recliners hours from 3 to 8 is the West incliner: onely remember, that as the stile in the West recliner beholds the North, and the plane the Zenith; so in the East incliner, the stile must behold the South, and the plane the Nadir.

Probl. 9.

To draw the hour-lines upon any direct South reclining or inclining plane.

AS the base of East and West reclining or inclining planes do alwayes lie in the meridian of the place, or pa-

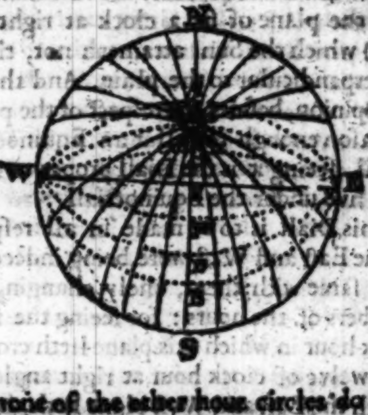
rallel thereunto, and the poles in the prime vertical; so doth the base of South and North reclining or inclining planes lie in the prime vertical or azimuth of East and West, and their poles consequently in the Meridian. Now if you suppose the circle of position, (which astrologically taken is fixed in the intersection of the meridian and horizon) to move about upon the horizon, till it comes into the plane of the prime vertical, and being fixed in the intersection thereof with the horizon, to be let fall either way from the Zenith upon the meridian; it shall truly represent all the South and North reclining and inclining planes also, of which there are six varieties three of South and three of North reclining; for either the South plane doth recline just to the pole, and then it becommeth an Equinoctiall, because the poles of that plane do then lie in the Equinoctiall; some call it a polar plane, or else it reclineth more and less then the pole, and consequently the poles of the plane above and under the Equinoctiall, somewhat differing from the former. In like manner, the North plane reclineth just to the Equinoctiall, which then becometh a polar plane, because the poles of that plane lie in the

poles

poles of the world; some term it an Equinoctiall plane. Or else it reclineth more or lesse then the Equinoctiall, and consequently the poles of the plane above and under the poles of the world, somewhat differing from the former.

Of the Equinoctiall plane.

The first of these six varieties which I call an Equinoctial plane, is in the fundamental Scheme, & also in this, represented by the six of clock hour-circle E P W, wherein you may observe out of the Scheme it self



that none of the other hour circles do cut the same, and therefore (as in the 5 Probl.)

you may conclude, that the hour-lines thereof have no center to meet in, but must be parallel one to another, as they were in the East and West Dialls.

And because this Diall is no other but the very horizontall of a right Sphere, where the Equinoctial is Zenith, and the Poles of the world in the Horizon; therefore it is not capable of the six of clock hour (no more then the East and West are of the 12 a clock hour) which vanish upon the planes, unto which they are parallel: and the twelve a clock hour is the middle line of this Diall (because the Meridian cutteth the plane of six a clock at right angles) which the Sun attaineth not, till he be perpendicular to the plain. And this in my opinion, besides the respect of the poles, is reason enough to call it an Equinoctiall Diall, seeing it is the Diall proper to them that live under the Equinoctiall.

This Diall is to be made in all respects as the East and West were, being indeed the very same with them, onely changing the numbers of the hours: for seeing the six of clock hour in which this plane lieth crosseth the twelve of clock hour at right angles, in which the East and West plane lieth, the rest of the hour-lines will have equall respect

unto

unto them both: so that the fifth hour from six of the clock is equall to the fift hour from twelve; the four to the four; and so of the rest. These analogies holding, the hour-distances from six are to be set off by the naturall tangents in these Dials, as they were from twelve in the East and West Dials.

The Geometricall Projection.

Draw the tangent line D S K, parallel to the line E Z W in the Scheme, crosse it at right angles with M S A the Meridian line, make S A the Radius to that tangent line, on which prick down the hours; and that there may be as many hours upon the plane as it is capable of, you must proportion the stile to the plane (as in the fifth Problem) after this manner: let the length of the plane from A be given in known parts, then because the extream hours upon this plane are 5 or 7, reckoning 15 degrees to every hour from 12, the arch of the Equator will be 75 degrees; and therefore in the right angled plain triangle S A ♎, we have given the base A ♎, the length of the plane from A, and the angle A S ♎ 75 degrees, to finde the perpendicular S A; for which, as in the fifth Chapter, I say;

As the Radius 90, 10,0000000
Is to the base A 20 3.50, 9,9410068
So is the tangent of A 12 d. 55. 9,4280052

To the perpendicular A S 94 3,9372120

At which height a stile being erected over the 12 a clock hour line, and the hours from

as drawn parallel thereunto through the points made in the tangent line, by setting off the natural tangents thereon, and then the Diall is finished.

Let S A 12 be placed in the meridian, and the whole plane at S raised to the height of the pole 51 degr. 53 min. then will the stile shew the hours truly, and the Diall stand in its due position.

2. *Of South reclining lesse then the pole.*

This plane is represented by the prickt circle in the fundamental Diagram E C W, and is intersected by the hour circles from the pole P, as by the Scheme appeareth, and therefore the Diall proper to this plane must have a center, above which the South pole is elevated, and therefore the stile must look downwards, as in South direct planes; to calculate which Dials there must be given the Poles elevation, and the quantity of reclination, by which to finde the hour distances from the meridian, thus in the triangle P C 1, having the poles elevation 51 degr. 53 min. and the reclination 25 degr. P C is given, by subtracting 25 degr. from P Z 38 degr. 47 min. the complement of the poles height, the angle C P 1 is 15 degrees, one hours distance, and

the angle at C right, we may finde C 1, by the first case of right angled spherical triangles: for,

As the Radius 90, 10,000000
Is to the sine of P C 13. 47. 9.367237
So is the tangent of C P 1. 15 9.428052
To the tangent of C 1 3.57. 8.795289

And this being all the varieties, save onely increasing the angle at P, I need not reiterate the work.

3. Of South reclining more then the pole.

This plane in the fundamental Scheme is represented by the prime circle EAW, of which in the same latitude let the reclination be 55 degrees, from which if you deduct P Z 38 deg. 47 min. the complement of the poles height, there will remain P A 16 deg. 53 min. the height of the north pole above the plane, and instead of the triangle P C 1, in the former plane we have the triangle P A 1, in which there is given as before the angle at P 15 deg. & the height of the pole P A 16 deg. 53 min. and therefore the same proportion holds: for,

As the Radius 90, 10,000000
Is to the sine of P A 16.53. 9.454108
So is the tangent of A 15. 9.428052
To the tangent of A 1. 4.36. 8.882160

The

The rest of the hours, as in the former, are thus computed, varying onely the angle at P.

The Geometricall Projection.

These angles being thus found, to draw the Dials true, consider the Scheme, wherein so oft as the plane falleth between Z and P, the Zenith and the North pole, the South pole is elevated; in all the rest the North; the substile is in them all the meridian, as in the direct North and South Dials; in which the stile and houres are to be placed, as was for them directed: which being done let the plane reclining lesse then the pole be raised above the horizon to an angle equal to the complement of reclination, which in our example is to 65 degr. and the axis of the plane point downwards; and let all planes reclining more then the pole have the hour of 12 elevated above the horizon to an angle equall to the complement of the reclination also, that is in our example to 35 deg. then shall the axis point up to the North pole, and the Diall lined on the plane with equall divisions, you have the houres desired. The houre-lines being drawn, erect a straight pin of wier upon the center, of ... as you please, and the Diall is finished.

Probl. 10.

To draw the hour-lines upon any direct North reclining or inclining plane.

THe direct north reclining planes have the same variety that the South had; for either the plane may recline from the Zenith just to the Equinoctial, and then it is a Polar plane, as I called it before, because the poles of the plane lie in the poles of the world; or else the plane may recline more or lesse then the Equinoctial, and consequently their poles do fall above or under the poles of the world, and the houre lines do likewise differ from the former.

Of the Polar plane.

This plane is well known to be a Circle divided into 24 equall parts, which may be done by drawing a circle with the line of Chords, and then taking the distance of 15 degrees from the same Chord, and drawing streight lines from the center through those equall divisions, you have the houre-lines desired. The houre-lines being drawn, erect a streight pin of wier upon the center, of what length you please, and the Diall is finished:

finished: yet seeing our Latitude is capable of no more then 16 houres and a halfe, the six houres next the South part of the Meridian, 11, 10, 9, 1, 2, and 3, may be left out as uselesse. Nor can the reclining face serve any longer then during the Suns aboad in the North part of the Zodiac, and the inclining face the rest of the year, because this plain is parallel to the Equinoctial, which the Sun crosseth twice in a year. These things performed to your liking, let the houre of 12 be placed upon the Meridian, and the whole plain raised to an angle equall to the complement of your Latitude, the which in this example is 38 deg. 47 min. so is this Polar plain and Diall rectified to shew the true houre of the day.

3. Of North reclining less than the Equator.

The next sort is of such reclining plains as fall between the Zenith and the Equator, and in the Scheme is represented by the pricked circle E F W, supposed to recline 25 degrees from the Zenith, which being added to P Z 38 deg. 47 min. the complement of the poles elevation, the aggregate is P F, 63 deg. 47 min. the height of the Pole or stile above the plane. And therefore

fore in the triangle P F 1, we have given P F, and the angle at P, to finde F 1, the said houres distance from the Meridian upon the plain, for which the proportion is,

As the Radius, 90,	10.000000
Is to the sine of P F, 63.47	9.951677
So is the tangent of F P 1, 15	9.428052
To the Tangent of F 1, 13.48	9.379739

In computing the other houre distances there is no other variety but increasing the angle at P as before we shewed.

3. *Of North reclining more then the Equator.*

The last sort is of such reclining plains as fall between the Horizon and Equator, represented in the fundamental Scheme by the prickt circle E B W, supposed to recline 70 deg. And because the Equator cutteth the Axis of the world at right angles, all planes that are parallel thereunto have the height of their stiles full 90 deg. above the plane: and by how much any plane reclineth from the Zenith, more then the Equator, by so much less then 90 is the height of the stile proper to it, and therefore if you adde P Z 38 deg. 47 min. the height of the Equator, unto Z B 70 deg. the reclination of

of the plain, the totall is P B 108 deg. 47 mi. whose complement to 180 is the arch B S, 71 deg. 53 min. the height of the pole above the plain. To calculate the houre lines thereof, we must suppose the Meridian P F B and the houre circles P 1, P 2, P 3, &c. to be continued till they meet in the South pole, then will the proportion be the same as before.

As the Radius, 90,	10.000000
To the sine of P B, 71.53	9.977033
So is the tangent of 1 P B, 15	9.428052
To the tangent of B 1, 14.27	9.405085

And so are the other houre distances to be computed, as in all the other planes.

The Gnometricall Projection.

The projection of these planes is but little differing from those in the last Probl. for the placing the houre and erecting the stile, they are the same, and must be elevated to an angle above the horizon equall to the complement of their reclinations, which in the North reclining lesse then the Equator is in our example 65 degrees, and in this plane the houres about the meridian, that is, from 10 in the morning till 2 in the afternoon, can never receive any shadow, by

reason

reason of the planes small inclination from the Zenith, and therefore needlesse to put them on. In the North reclining more then the Equator, the plaine in our example must be elevated 125 degr. above the horizon, and the stiles of both must point to the North pole.

Lastly, as all other planes have two faces respecting the contrary parts of the heavens; so these recliners have opposite sides, look downwards the Nadir, as those do towards the Zenith, and may be therefore made by the same rules; or if you will spare that labour, and make the same Dials serve for the opposite sides, turn the centers of the incliners downwards, which were upwards in the recliners; and those upwards in the incliners which were downwards in the recliners, and after this conversion, set the houres on the right hand of the meridian in the recliner, houres on the left hand in the incliner, and contrarily; so have you done what was desired: and this is a general rule for the opposite sides of all planes.

Probl. [illegible]

To draw the hour-lines upon a declining reclining, or declining inclining plane.

DEclining reclining planes have the same varieties that were in the former reclining North and South; for either the declination may be such, that the reclining plane will fall just upon the pole, and then it is called a declining Equinoctial; or it may fall above or under the pole, and then it is called a South declining east and west recliner: on the other side the declination may be such, that the reclining plane shall fall just upon the intersection of the Meridian and Equator; and then it is called a declining polar; or it may fall above or under the said intersection, and then it is called a North declining East and West recliner. The three varieties of South recliners are represented by the three circles, A H B falling between the pole of the world and the Zenith; A G B just upon the pole; and A E B between the pole and the horizon: and the particular pole of each plane is so much elevated above the horizon, (upon the azimuth)

zimuth D Z C, crossing the base at right angles) as the plane it self reclines from the Zenith, noted in the Scheme, with I, K, and L.

1. Of the Equinoctiall declining and reclining plane.

This plane represented by the circle A G B, hath his base A Z B declining 30 degrees from the East and West line E Z W equal to the declination of the South pole thereof 30 degrees from S the South part of the Meridian Easterly unto D, reclining from the Zenith upon the azimuth C Z D the quantity Z G 34 degrees, 53 min. and passeth

passeth through the pole at P. Set off the reclination Z G, from D to K, and K shall represent the pole of the reclining plane so much elevated above the horizon at D, as the circle A G B representing the plane declineth from the Zenith Z, from P the pole of the world, to K the pole of the plane, draw an arch of a great circle P K, thereby the better to informe the fancie in the rest of the work. And if any be desirous, to any declination given, to fit a plane reclining just to the pole: or any reclination being given, to finde the declination proper to it, this Diagram will satisfie them therein: for in the Triangle Z G P, we have limited,

First, the hypothenusal P Z 38 degrees, 47 min.

Secondly, the angle at the base P Z G, the planes declination 30 degrees. Hence to finde the base G Z, by the seventh case of right angled spherical triangles, the proportion is;

As the Radius 90,	10,000000
To the co-sine of GZP 30;	9.937531
So the tangent of PZ 38.47.	9.900138
To the tangent of GZ 34.53.	9.837669

the reclination required.

If the declination be required to a recli-
nation given, then by the 13 case of right
angled spherical triangles, the proportion is

As the Radius 90,	10.000000
To the tangent of ZG 34.53.	9.837669
So the co-tangent of PZ 38.47.	10.099861
To the co-sine of GZP 39.	9.937530

And now to calculate the hour-lines of this Diall, you are to finde two things: first, the arch of the plane, or distance of the meridian and substile from the horizontal line, which in this Scheme is PB, the intersection of the reclining plane with the horizon, being at B. And secondly, the distance of the meridian of the place SZPN, from the meridian of the plane PK, which being had, the Diall is easily made.

Wherefore in the triangle ZGP, right angled at G, you have the angle GZP given 30 degrees, the declination; and ZP 38 degr. 47 min. the complement of the Pole; to finde GP: and therefore, by the eighth case of right angled spherical triangles, the proportion is:

As

As the Radius, 90 — 10.000000
To the sine of Z P, 38.47 — 9.795863
So is the sine of G Z P, 30 — 9.698970
To the sine of G P, 18.12 — 9.492833

Whose complement 71 deg. 48 min. is the arch P B desired.

The second thing to be found is the distance of the Meridian of the place, which is the houre of 12 from the substile or meridian of the plane, represented by the angle Z P G, which may be found by the 11 Case of right angled sphericall Triangles, for

As the Radius, 90 — 10.000000
Is to the sine of G P, 18.12 — 9.492833
So is the co-tang. of G Z, 34.53 — 10.162379
To the co-tang. of G P Z, 65.48 — 9.655212

Whose complement is Z P K 24 deg. 12 min. the arch desired.

Now because 24 deg. 12 min. is more then 15 deg. one houres distance from the Meridian, and lesse then 30 deg. two houres distance, I conclude that the stile shall fall between 10 and 11 of the clock on the West side of the Meridian, because the plain declineth East; if then you take 15 deg. from 24 deg. 12 min. there shall remain 9 deg. 12 min. for the Equinoctiall distance of the

11 aclock

11 aclock houre line from the subſtile, and taking 24 deg. 32 min. out of 30 deg. there ſhall remain 5 deg. 68 min. for the diſtance of the houre of 16 from the ſubſtile; the reſt of the houre diſtances are eaſily found by continual addition of 15 deg. Unto theſe houre diſtances joyn the naturall tangents as in the Eaſt and Weſt Dials, which will give you the true diſtāces of each houre from the ſubſtile, the plane being projected as in the 5 Pro. for the eaſt & weſt dials, or as in the 8 Prob. for the Equinoctial, according to which rules you may proportion the length of the ſtile alſo, which being erected over the ſubſtile, and the Diall placed according to the declination 30 deg. eaſterly, and the whole plain raiſed to an angle of 55 deg. 47 min. the complement of the reclination, the ſhadow of the ſtile ſhall give the houre of the day deſired.

5. *To draw the houre lines upon a South reclining plain, declining Eaſt or Weſt, which paſſeth between the Zenith and the Pole.*

In theſe kinde of declining reclining plains, the South pole is elevated above the plane, as is clear by the circle A H B repreſenting the ſame, which falleth between the Zenith and the North pole, and therefore

fore hideth that pole from the eye, and forceth you to seeke the elevation of the contrary pole above the plain, which notwithstanding maketh the like and equall angles upon the South side objected to it, as the North pole doth upon the North side, (as was shewed in the 7 Prop.) so that either you may imagine the Scheme to be turned about, and the North and South points changed, or you may calculate the houres as it standeth, remembring to turn the stile upwards or downwards, and change the numbers of the houres, as the nature of the Diall wil direct you.

In this sort of declining reclining Dials, there are four things to be sought before you can calculate the houres.

1 The distance of the Meridian from the Horizon.

2 The height of the pole above the plain.

3 The distance of the substile from the Meridian.

4 The angle of inclination between the Meridian of the plane, and the meridian of the place.

1 The distance of the Meridian from the Horizon, is represented by the arch O B, to finde which, in the right angled Triangle H O Z, we have H Z the reclination 20 deg. and

and the angle H Z O the declination, to find H O, the complement of O B, for which, by the first case of right angled sphericall triangles, the analogie is,

As the Radius, 90	10.000000
To the sine of H Z, 20	9.534051
So is the tangent of H Z O, 30	9.761439
To the tangent of H O, 77.17	9.295490

Whose complement 78 deg. 83 min. is O B, the arch desired.

3. To finde the height of the pole above the plane, there is required two operations, the first to finde O P, and the second to finde P R; O P may be found by the 3 Case of right angled Sphericall Triangles, for,

As the Radius, 90	10.000000
Is to the co-sine of H Z P, 30	9.937531
So is the co-tang. of H Z, 20.	10.438934
To the co-tangent of Z O, 22.80	10.376465

Which arch being found, and deducted out of Z P 38 deg. 47 min. there resteth P O 15 deg. 67 min.

Then may you finde P R, by the triangles H Z O, and P R O, both together, because the sines of the hypotheneusals, and the sines of the perpendiculars, are proportional, by the first of the 7 Chap. of Triangles,

b 2a There-

Therefore,

As the sine of Z O, 21.80	9.588289
Is to the sine of Z H, 20	9.534052
So is the sine of P O, 15.67	9.431519
To the sine of P R, 13.79	9.377282

The height of the stile desired.

3 The distance of the substile from the Meridian may be found by the 12 Case of right angled sphericall triangles, for

As the co-sine of P R, 13 78	9.987298
Is to the Radius, 90	10.000000
So is the co-sine of P O, 15.67	9.983551
To the co-sine of O R, 7.41	9.996253

The arch desired.

4. The angle of inclination between the Meridians, may be found by the 11 Case of right angled Spherical triangles, for,

As the Radius, 90	10.000000
Is to the sine of P R, 13.79.	9.377241
So is the co-tang. of OR 7.51	10.879985
To the co-tang. of OPR, 28.93	10.257226

Now as in all the former works, the angle P between the two Meridians being 28 deg. 93 min. which is more then one Houres distance from the Meridian, and lesse then

two, you may conclude that the subſtile muſt ſtand between the firſt & ſecond hours from the Meridian or 12 of the clock Weſterly, becauſe the declination is eaſterly: and 28 deg. 93 min. being deducted out of 30 deg. there reſteth 1 deg.7 min. for the diſtance of 10 of the clock from the ſubſtile; again, deducting 15 deg. from 28 deg. 93 min. there reſteth 13 deg. 73 min. the diſtance of the 11 aclock houre line from the ſubſtile, the reſt are found by continuall addition of 15 deg. as before.

And here the true houre diſtances may be found by the firſt caſe of right angled Sphericall triangles, for,

As the Radius, 90	10.000000
Is to the ſine of P R 13.79	9.377240
So is the tangent of R P, 11.15	9.428051
To the tangent of R 11, 3.66	8.805292

And ſo proceed with all the reſt.

3. *To draw the houre lines upon a South reclining plain, declining Eaſt or weſt, which paſſeth between the Pole and the Horizon.*

In this plain repreſented by the circle of reclination A F B, the North pole is elevated above the plane, as the South po-

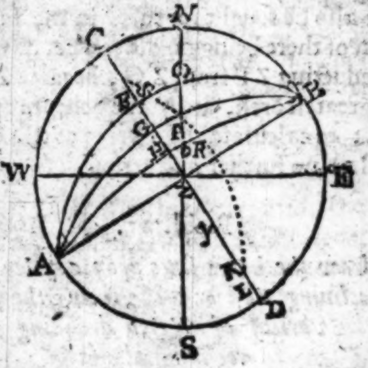
wa-

was above the other, and the same four things that you found for the former Diall must also be sought for this; in the finding whereof there being no differerice, save only deductiug Z P from Z O, because Z O is the greatest arch, as by the Scheam appeareth: to calculate the houres of this plane needeth no further instruction.

Probl. 12.

To draw the houre lines upon a polar plain, declining East or West, being the first variety of North declining reclining planes

AS in the South declining recliners, there are three varieties, so are there in the North as many: for either the plane reclining doth passe by the intersection of the Meridian and Equator, and then it is called a declining Polar, which hath the substile alwayes perpendicular to the Meridian; or else it passeth above or under the intersection of the Meridian and Equator, which somewhat differeth from the former. I will therefore first shew how they lie in the Scheam, and then proceed to the particular making of the Dials proper to them.



1. *Of the Polar declining reclining plane.*

This plane is in this diagram represented by the circle A G B, Z G is the reclination, Z Æ the distance of the Equator from the Zenith, the declination N C, K the pole thereof. Here also as in the last Probl. there may be a reclination found to any declination given, and contrary, by which to fit the plane howsoever declining, to passe through the intersection of the Meridian and Equator, by the 7 and 13 Cases of right-angled sphericall triangles.

As

As the Radius, 90	10.000000
To the co-sine of O Z Æ, 60	9.698970
So is the tangent of Z Æ, 51.53	10.099861
To the tangent of Z G, 32.18	9.798871

The reclination desired.

And,

As the Radius, 90	10.000000
To the tangent of G Z, 32,18	9.798831
So is the co-tangent of Z Æ, 51.53	9.900138
To the co-sine of GZ Æ, 60	9.698969

The declination.

And now to calculate the houre lines of this Dial, you must finde, first, the distance of the Meridian from the Horizon, by the 8 Case of right angled spherical triangles.

As the Radius, 90	10.000000
Is to the sine of Z Æ, 51.53	9.893725
So is the sine of G Z Æ, 60	9.937531
To the sine of Æ G, 42.69	9.831256

Whose complement 47 deg. 31 min. is A Æ the arch desired.

2. You must finde R P, the height of the pole above the plane, by the 2 Case of right angled Sphericall Triangles, for

As the Radius, 90	10.000000
Is to the sine of Æ Z G, 60	9.937531
So is the co-sine of Z G, 32.11	9.927565
To the co-sine of Z Æ G, 42.87	9.865096

 Which

Which is the height of the pole above the plane, Æ R being a Quadrant, PR must needs be the measure of the angle at Æ.

3. Because in all decliners (whose planes passe by the intersection of the Meridian and Equinoctiall) the substile is perpendicular to the Meridian, therefore you need not seek Æ R, the distance between the substile and Meridian, which is alwayes 90 deg. and falleth upon the 6 a clock houre.

4. Lastly, the arch Æ R, which is the distance of the substile from the Meridian: being 90 degrees, the angle at P opposite thereunto must needs be 90 also: from whence it followes, that the houres equidistant from the six of the clock hour in Equinoctial degrees shall also have the like distance of degrees in their arches upon the plane, and so one half of the Diall being calculated, serves for the whole; these things considered, the true hour-distances may be found, by the first case of right angled spherical triangles: for,

As the Radius,	10.000000
Is to the sine of P R 42.87.	9.832724
So is the tangent of RP 5. 15 d.	9.428052
To the tangent of R 5 10,34.	9.260776

The

The which 10 degr. 34 min. is the true distance of 5 and 7 from the substile or six of the clock hour, and so of the rest.

The Geometrical projection of this plane needs no direction; those already given are sufficient, according to which this Diall being made and rectified by the declination and reclination given, it is prepared for use.

3. To draw the hour-lines upon a North reclining plane, declining East or west, which cutteth the meridian between the Zenith and the Equinoctial.

All North reclining planes howsoever declining, have the North pole elevated above them, and therefore the center of the Diall must be so placed above the plane, that the stile may look upwards to the pole, neither can it be expected that the plane being elevated above the horizon Southward, should at all times of the year be enlightened by the Sun, except it recline so far from the Zenith, as to intersect the Meridian between the horizon and the Tropique of *Capricorn*; this plane therefore reclining but 16 degrees from the Zenith, and declining 60 cannot shew many hours, when the Sun

is in his greatest Northern declination, partly by reason of the height of the plane above the horizon, and partly by reason of the great declination thereof, hindring the Sun-beams from all the morning houres, which may be therefore left out as uselefs.

In this second variety, the plane represented by the Circle A M B in the last Diagram, cutteth the Meridian at O between the Zenith and the Equator, Z M being the reclination, 16 deg. Z Æ the distance of the Equator from the Zenith, 51 deg. 53 m. and the declination N C 60 as before.

As in the former, so in this Diall, the same four things are again to be found before you can calculate the houre distances thereof. The first is the distance of the Meridian from the Horizon, represented in this plain by the arch A M. The second is P R, the height of the pole above the plane. The third is N R, the distance of the substile or Meridian of the plane, from the Meridian of the place. The fourth is the angle MPR between the two Meridians: all which, and the houre distances also, being to be found according to the directions of the last Probl. there needeth no further instruction here.

3 To

3 *To draw the houre lines upon a North reclining plane, declining East or West, which cutteth the Meridian between the Equator and Horizon.*

The last variety of the six declining recliners, represented by the circle A L B, and cutteth the Meridian at H, between the Equator and the Horizon, Z L being the reclination, 54 deg. the declination N C, 60 deg. as before; and hence the four things mentioned before must be sought ere you can calculate the houre distances.

1 The distance of the Meridian and Horizon, represented by A H.

2 R H the substile or Meridian of the plane from the Meridian of the place.

3 P R, the height of the pole above the plane.

4 H P R, the angle between the two Meridians. In finding whereof the proportions are still the same, though the triangles are somewhat altered, for when you have found Z H, it is to be added to Z P to finde P H, both which together do exceed a Quadrant, therefore the sides P N must be continued to X, then is P X the complement of P H to a semicircle, and if R B be continued

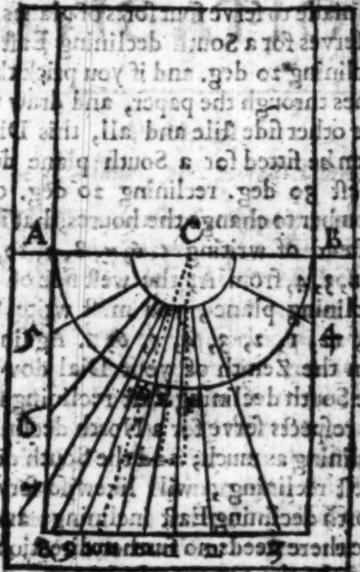
o X also, R X may be found by the 12 Case of right angled spherical triangles as before, whose complement is R H, the distance of the substile from the Meridian; and hence the angle at P must be found in that triangle also, though the proportion be the same, there being no other variety, I think it needlesse to reiterate the work.

The Geometrical Projection.

There is so little difference between the South & North declining reclining planes, that the manner of making the Dials for both may be shewed at once: Let the example therefore be a Diall for a South plane declining East 30 deg. reclining 20 deg.

First, draw the horizontal line A C B in the middle of the plane, because the stile of this Dial must look downwards to the South pole, and because the plane declineth East, therefore the morning houres must stand on the West side of the Meridian, and the distance of the Meridian and Horizon 78 deg. 83 min. must be set upon the circle A D B F, from A to E, and there draw the line C E for the 12 a clock houre, from E reckon 7 deg. 51 min. the distance of the substile and Meridian Westwards to D, and draw the prickt line C D for the substile: from the

the point of the subſtile at D, ſet off the houre diſtances, as of 3 deg. 66 min. for 11, and ſo of the reſt: unto every prick draw ſtreight lines from the center C, ſo have you



all the houres truly drawn. Laſt of all, ſet off from D the height of the ſtile D E, and draw the line C E for the axis, which being erected

erected over the subſtile, C D, the Diall is fit for uſe, but muſt be placed in its due poſition by the declination and reclination thereof.

And thus have you made four Dials at once, or at leaſt, this Dial thus drawn may be made to ſerve four ſorts of planes, for firſt, it ſerves for a South declining Eaſt 30 deg. reclining 20 deg. and if you prick the houre lines through the paper, and draw them on the other ſide ſtile and all, this Diall will then be fitted for a South plane declining Weſt 30 deg. reclining 20 deg. only remember to change the houres, that is to ſay, inſtead of writing 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, from A, the weſt ſide of the Eaſt declining plane, you muſt write, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, 7. Again, if you turn the Zenith of your Dial downwards, the South declining Eaſt reclining ſhall in all reſpects ſerve for a North declining weſt inclining as much; and the South declining Weſt reclining, will likewiſe ſerve for a North declining Eaſt inclining; and therefore there needs no further direction either to make the one, or calculate the other.

CHAP.

CHAP. III.

Of the Art of NAVIGATION.

Probl. 1.

Of the 32 windes, or Seamans Compaß.

THe course of a ship upon the Sea dependeth upon the windes; The designation of these depends upon the certain knowledge of one principal; which considering the situation, and condition of the whole Sphere, ought in nature to be North or South, the North to us upon this side of the Line, the South to those in the other hemisphere; for in making this observation men were to intend themselves towards one fixed part of the heavens or other, and therefore to the one of these. In the South part there is not found any star so notable, and of so neer distance from the Pole, as to make any precise or firm directi-

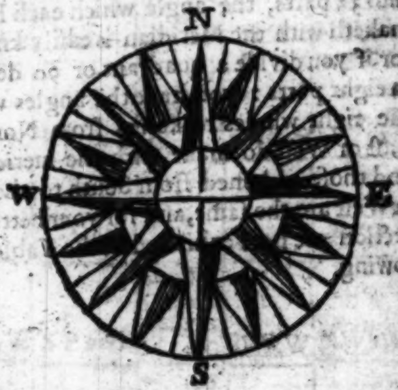
on

on of that winde, but in the North we have that of the second magnitude in the taile of the lesser Bear, making so small and incensible a circle about the Pole, that it commeth all to one, as if it were the Pole it self. This pointed out the North winde to the Mariners of old especially, and was therefore called by some the Lead or Lead star; but this could be only in the night, and not alwayes then. It is now more constantly and surely shewed by the Needle touched with the Magnet, which is therefore called the Load or Lead-stone, for the same reason of the leading and directing their courses to the North and South position of the earth, not in all parts directly, because in following the constitution of the great Magnet of the whole earth, it must needs be here and there led aside towards the East or West by the unequal temper of the Globe, consisting more of water then of earth in some places, and of earth more or lesse Magnetical in others.

This deviation of the Needle, the Mariners call North-easting, & North-westing, as it falleth out to be, otherwise, and more artificially, the *Variation of the Compaß*, which though it pretend uncertainty, yet proveth to be one of the greatest helps the

Sea-man

Seaman hath. And the North and South windes being thus assured by the motion either of direction or variation of the needle, the Mariner supposeth his ship to be (as it alwayes is) upon some Horizon or other, the center whereof is the place of the ship.



The line of North and South found out by the Needle, a line crossing this at right angles, sheweth the East and West, and so they have the four Cardinall windes, crosse again each of these lines, and they have the eight whole windes, as they call them. Another division of these maketh eight more which they call halfe windes, a third maketh 16, which they call the quarter windes, so

ſo they are 32 in all. Every one of theſe Windes is otherwiſe termed a ſeveral point of the Compaſſe, and the whole line conſiſting of two windes, as the line of North and South, or that of Eaſt and Weſt is called a Rumb. The Windes and Rumbs thus aſſigned by an equal diviſion of a great Circle into 32 parts, the angle which each Rumb maketh with the Meridian is eaſily known, for if you divide a quadrant or 90 degrees in eight parts: you have the angles which the eight windes reckoned from North to Eaſt or Weſt do make with the meridian; and thoſe reckoned from South to the Eaſt or Weſt are the ſame, and for your better direction are here exhibited in the Table following.

A Table for the angles which every Rumbe maketh with the Meridian.

North	*South*	D. part	*South*	*North*
		02.8125		
		05.6250		
		08.4375		
N *by* E	S *by* E	11.2500	S *by* W	N *by* W
		14.0625		
		16.8750		
		19.6875		
N N E	S S E	22.5000	S S W	N N W
		25.3125		
		28.1250		
		30.9375		
N E *by* N	S E *by* S	33.7500	S W *by* S	N W *b* N
		36.5625		
		39.3750		
		42.1875		
N E	S E	45.0000	S W	N W
		47.8125		
		50.6250		
		53 4375		
N E *by* E	S E *by* S	56.2500	S W *b* W	N W *b* W
		59.0625		
		61.8650		
		64.6875		
E N E	E S E	67.5000	W S W	W N W
		70.3125		
		73.1250		
		75.9375		
E *by* N	E *by* S	78.7500	W *by* S	W *by* N
		81.5625		
		84.3750		
		87.1875		
East	*East*	90.0000	*West*	*West*

Probl. 2.

Of the description and making of the Sea-chart.

THe Sea-mans Chart is a Parallelogram, divided into little rectangled figures, and in the plain Chart are equal Squares, representing the Longitudes and Latitudes of such places, as may be set in the Chart, but the body of the earth being of a Globular form, the degrees of Longitude reckoned in the Equator from the Meridian, are in no place equal to those of the Latitude reckoned in the Meridian from the Equator, save onely in the Equinoctial; for the degrees of latitude are all equall throughout the whole Globe, and as large as those of the Equinoctial; but the degrees of Longitude at every parallel of latitude lessen themselves in such proportion as that parallel is lesse then the Equinoctial: This dis-proportion of longitude and latitude caused for a long time much errour in the practise of Navigation, till at last it was in part reconciled by *Mercator*, that famous Geographer: and afterwards exactly rectified by our worthy Countreyman Master *Edward Wright*, in his Book entituled, *The Cor-*

Correction of Errours in Navigation: In which he hath demonstrated by what proportion the degrees of Longitude must either increase or decrease in any Latitude, his words are as followeth.

Suppose, saith he, a spherical Superficies, with Meridians, Parallels, Rumbes, and the whole Hydrographial description drawn thereupon, to be inscribed into a concave Cylinder, their axes agreeing in one.

Let this Spherical superficices swell like a bladder (whiles it is in blowing) equally alwayes in every part thereof (that is as much in Longitude as in Latitude, till it apply and joyn it self (round about and all along also towards either pole) unto the concave superficies of the Cylinder: each parallel upon this spherical superficies increasing successively from the Equinoctial towards either pole, until it come to be of equal diameter with the Cylinder, and consequently the Meridians, stil inclining themselves, till they come to be so far distant every where each from other, as they are at the Equinoctial.

Thus it may most easily be understood, how a spherical superficies may by extension be made a Cylindrical, and consequently a plain parallelogram superficies: because

cause the superficies of a cylinder is nothing else but a plain parallelogram wound about two equal equidistant circles, that have one common axletree perpendicular upon the centers of them both, and the peripheries of each of them equall to the length of the parallelogram, as the distance betwixt those circles, or height of the cylinder is equall to the bredth thereof, So as the Nautical planisphere may be defined to be nothing else but a parallelogram made of the Spherical superficies of an Hydrographical Globe inscribed into a concave cylinder, both their axes concurring in one, and the sphericall superficies swelling in every part equally in longitude and latitude, till every one of the parallels thereupon be inscribed into the cylinder (each parallel growing as great as the Equinoctial, or till the whole spherical superficies touch and apply it self every where to the concavity of the cylinder.)

In this Nautical planisphere thus conceived to be made, all places must needs be situate in the same longitudes, latitudes, and directions or courses, and upon the same meridians, parallels and rumbes, that they were in the Globe, because that at every point between the Equinoctial and the Pole

we understand the spherical superficies, whereof this plan sphere is conceived to be made, to swell equally as much in longitude as in latitude (till it joyn it self unto the concavity of the cylinder; so as hereby no part thereof is any way distorted or displaced out of his true and natural situation upon his meridian, parallel or rumbe, but onely dilated and enlarged, the meridians also, parallels, and rumbes, dilating and enlarging themselves likewise at every point of latitude in the same proportion.

Now then let us diligently consider of the Geometrical lineaments, that is, the meridians, rumbes, and parallels of this imaginary Nautical planisphere, that we may in like manner expresse the same in the Mariners Chart: for so undoubtedly we shall have therein a true Hydrographical description of all places in their longitudes, latitudes, and directions, or respective situations each from other, according to the points of the compasse in all things correspondent to the Globe, without either sensible or explicable errour.

First, therefore in this planisphere, because the parallels are every where equal each to other (for every one of them is equal to the Equinoctiall or circumference of the

cir-

circumscribing cylinder) the meridians also must needs be parallel & streight lines; and consequently the rumbes, (making equall angles with every meridian) must likewise be streight lines.

Secondly, because the spherical superficies whereof this planisphere is conceived to be made, swelleth in every part thereof equally, that is as much in Latitude as in Longitude, till it apply it self round about to the concavity of the cylinder: therefore at every point of Latitude in this planisphere, a part of the Meridian keepeth the same proportion to the like part of the parallel that the like parts of the Meridian, and parallel have each to other in the Globe, without any explicable errour.

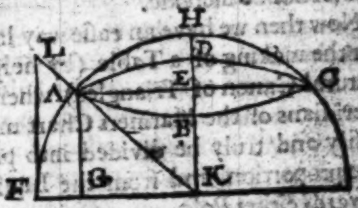
And because like parts of wholes keep the same proportion that their wholes have therefore the like parts of any parallel and Meridian of the Globe, have the same proportion, that the same parallel and meridian have.

For example sake, as the meridian is double to the parallel of 60 degrees: so a degree of the meridian is double to a degree of that parallel, or a minute to a minute, and what proportion the parallel hath to the meridian, the same proportion have

their

their diameters and ſemidiameters each to other.

But the ſine of the complement of the parallels latitude, or diſtance from the Equinoctial, is the ſemidiameter of the parallel.



As here you ſee A E, the ſine of A H, the complement of A F, the latitude or diſtance of the parallel A B C D from the Equinoctial, is the ſemidiameter of the ſame parallel. And as the ſemidiameter of the meridian or whole ſine, is to the ſemidiameter of the parallel; ſo is the ſecant or hypothenuſa of the parallels latitude, or of the parallels diſtance from the Equinoctial, to the ſemidiameter of the meridian or whole ſine; as F K, (that is A K) to A B (that is G K) ſo is L K, to K F.

Therefore in this nautical planiſphere, the

the Semidiameter of each parallel being equal to the semidiameter of the Equinoctial, that is, to the whole sine; the parts of the Meridian at every point of Latitude must needs increase with the same proportion wherewith the secants of the ark, conteined between those points of Latitude and the Equinoctial do increase.

Now then we have an easie way laid open for the making of a Table (by help of the natural Canon of Triangles) whereby the meridians of the Mariners Chart may most easily and truly be divided into parts, in due proportion, and from the Equinoctial towards either Pole.

For (supposing each distance of each point of latitude, or of each parallel from other, to contein so many parts as the secant of the latitude of each point or parallel conteineth) by perpetual addition of the secants answerable to the latitudes of each point or parallel unto the summe compounded of all the former secants, beginning with the secant of the first parallels latitude, and thereto adding the secant of the second parallels Latitude, and to the summe of both these adjoyning the secant of the third parallels Latitude; and so forth in all the rest we may make a Table which shall

shall truly shew the sections and points of latitude in the Meridians of the Nautical Planisphere, by which sections the parallels must be drawn.

As in the Table of meridional parts placed at the end of this Discourse, we made the distance of each parallel from other, to be one minute or centesm of a degree; and we supposed the space between any two parallels, next to each other in the Planispere, to contain so many parts as the secant answerable to the distance of the furthest of those two parallels from the Equinoctial; and so by perpetual addition of the secants of each minute or centesm to the sum compounded of all the former secants, is made the whole Table.

As for example, the secant of one centesm in Master *Briggs*'s *Trigonometrica Britannica* is 100000.00152, which also sheweth the section of one minute or centesm of the meridian from the Equinoctial in the Nautical Planisphere; whereunto adde the secants of two minutes or centesmes, that is 100000.00609, the sum is 200000.00761, which sheweth the section of the second minute of the meridian from the Equinoctial in the planisphere; to this sum adde the secant of three minutes, which is 100000.01371, the

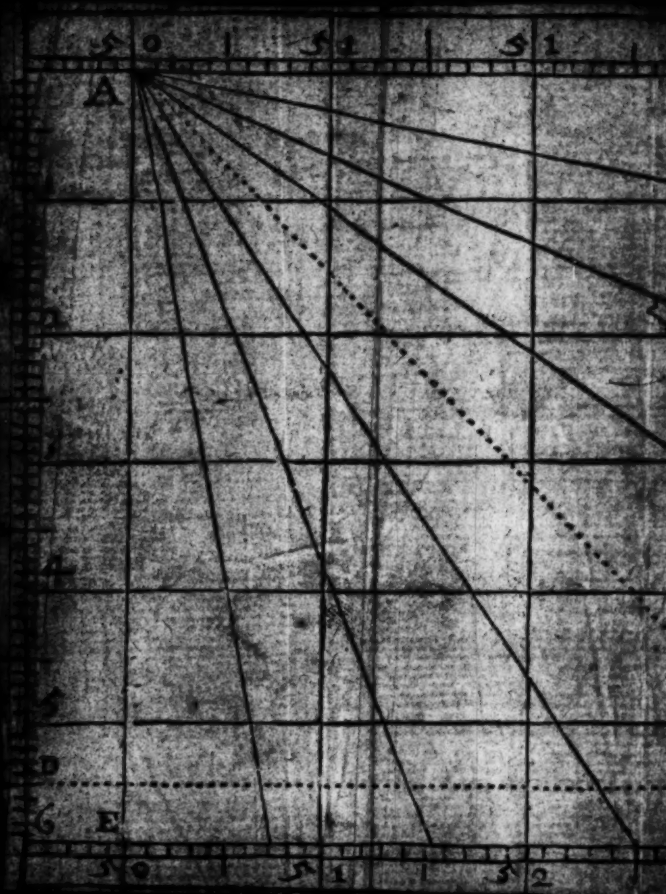
sum

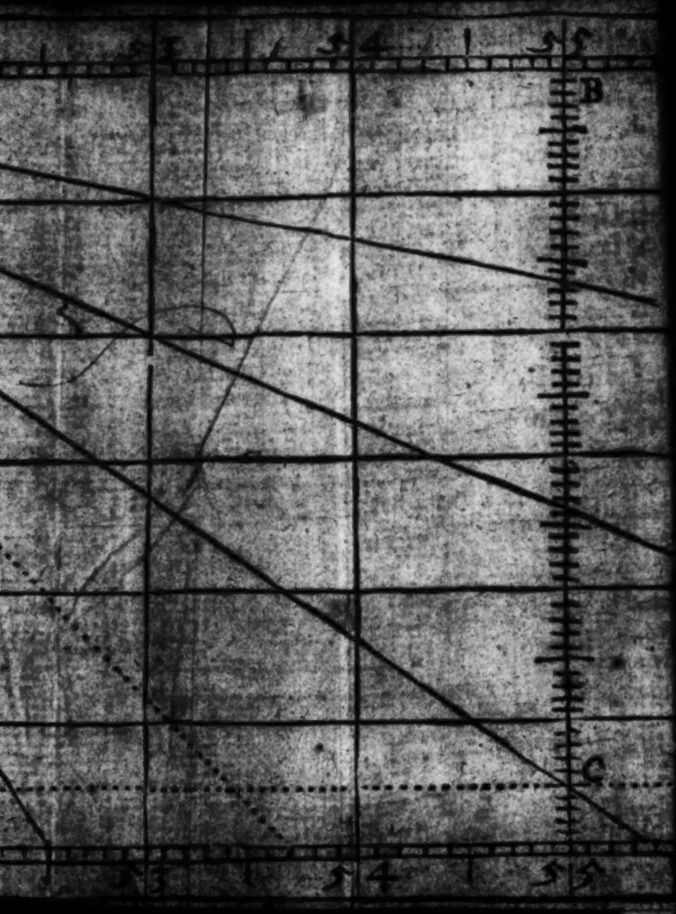
fum will be 3000 0.08131, which sheweth the section of the third minute of the me-ridian from the Equinoctial, and so forth in all the rest; but after the Table was thus finished, it being too large for so small a Volume, we have contented our selves with every tenth number, and have also cut off eight places towards the right hand, so that in this Table the section of 10 mi-nutes is 100, of one degree 1000, and this is sufficient for the making either of the ge-nerall or any particular Chart.

I call that a general Chart, whose line A E in the following figure represents the Equinoctial, (as here it doth the parallel of 50 degrees) and so containeth all the pa-rallels successively from the Equinoctial to-wards either Pole, but they can never be extended very near the Pole, because the di-stances of the parallels increase as much as secants do. But notwithstanding this, it may be termed general, because a more general Chart cannot be contrived *in plano*, except a true projection of the Sphere it self.

And I call that a particular Chart which is made properly for one particular Navi-gation; as if a man were to sail between the Latitude of 50 and 55 degrees, and his difference of Longitude were not to exceed

six


A
D
E



3
4
5
B
C

six degrees, then a Chart made, as this figure is for such a Voyage, may be called particular, and is thus to be projected.

Having drawn the line A B, serving for the first meridian, crosse it at right angles with the two perpendiculars B C and A E, divide the line A E, or another line parallel to it into six equal parts, noting them with 1, 2, 3, 4, 5, 6; then sub-divide each part or degree into 10, and if you can, each of those into 10 more; however, we suppose each degree to be subdivided into 1000 parts; through each of these degrees draw lines parallel to the first meridian A B. The meridians being drawn, to draw the parallels of latitude you must have recourse to your Table of meridionall parts, in which finding that the distance between the Equator and 50 degrees in the meridian should be equal to 57 degr. 909 parts in the Equator and his parallels; I may suppose the lowest parallel to be 57 degrees from the Equator. So the distance between the lowest parallel and the parallel of 50 degrees will be 909 parts onely; wherefore I take these 909 parts out of the line A E, and set them from the lowest parallel upwards, and draw the line A E, which shall represent the parallel of 50 degrees. In like manner I

S 2 that

finde by the Table that the distance between the Equator and 51 degrees in the meridian is 59 degrees, 981 parts; I abate the former 57 degrees, and there remains 2 degr. 481 parts, to be set from the lowest parallel upwards, by which to draw the parallel of 51 degrees; and so may the other parallels be also drawn.

Probl. 3.

The Latitudes of two places being known, to finde the Meridional difference of the same Latitudes.

IN this Proposition there are three varieties: First, when one of the places is under the Equinoctial, and the other without, and in this case the degrees and minutes in the Table answering to the latitude of that other place are the meridional difference of those Latitudes.

So if one place propounded were the entrance of the River of the *Amazones*, which hath no latitude at all, and the other the *Lizard*, whose latitude is 50 degrees, their difference will be found 57, 905.

2. When both the places have Northerly or Southerly Latitude, in this case if you substract the degrees and minutes in the Table

Table answering to the lesser Latitude, out of those in the same Table answering to the greater Latitude, the remainer will be the Meridional difference required.

Example.

Admit the Latitude of S. *Christophers* to be 15 deg. 50 parts or minutes, and the Latitude of the *Lizard* to be 50 degrees. In the Table of Latitudes, the number answering to

15 deg. 50 min. is 11,692
50 deg. 10 [illegible] 57,805
Their difference 42,113

3. When one of the places have Southerly and the other Northerly Latitude; in this case, the sum of the numbers answering to their Latitudes in the Table, is the meridional difference you look for.

So *Caput bonæ spei*, whose latitude is about 36 deg. 50 parts, and *Japan* in the *East Indies*, whose latitude is about 30 degrees being propounded, their meridional difference will be found to be 70,724.

For the meridional parts of 36. 50. 39,472
And the meridional parts of 30 d. 31,472
Their sum is the difference required. 70,724

Probl. 4.

Two places differing onely in Latitude, to finde their distance.

IN this proposition there are two varieties.

1. If the two places propounded lie under the same meridian, and both of them on one side of the Equinoctial, you must substract the lesser latitude from the greater, and the remainer converted into leagues, by allowing 20 leagues to a degree, will be the distance required.

2. If one place lie on the North, and the other on the South side of the Equinoctial (yet both under the same meridian) you must then adde both the latitudes together, and the sum converted into leagues, will give their distance.

Probl. 5.

Two places differing onely in longitude being given, to finde their distance.

IN this proposition there are also two varieties.

1. If the two places propounded lie under the Equinoctial, then the difference of their Longitudes reduced into leagues (by al-

allowing 20 leagues to a degree) giveth the distance of the places required.

3. But if the two places propounded differ onely in longitude, and lie not under the Equinoctial, but under some other intermediate parallel between the Equinoctial and one of the poles: then to finde their distance, the proportion is,

As the Radius,
Is to the co-sine of the common latitude;
So is the sine of half the difference of longitude,
To the sine of half their distance.

Probl. 6.

Two places being given, which differ both in Longitude and Latitude, to finde their distance.

IN this Proposition there are three varieties.

1. If one place be under the Equinoctial circle, and the other towards either pole, then the proportion is,

As Radius,
To the cosine of the difference of longitude;
So is the co-sine of the latitude given,
To the co-sine of the distance required.

2. If both the places propounded be without the Equinoctial, and on the Northern or Southern side thereof, then the proportion must be wrought at two operations.

1. Say; As the Radius,
To the cosine of the difference of Longitude
So the co-tangent of the lesser latitude,
To the tangent of the fourth ark.

Which fourth ark substract out of the complement of the greater latitude, and retaining the remaining ark say,

As the co-sine of the ark found,
Is to the co-sine of the ark remaining;
So is the sine of the lesser latitude,
To the co-sine of the distance required.

3. If the two places propounded differ both in Longitude and Latitude, and be both of them without the Equinoctial, and one of them towards the North pole, and the other towards the South pole, the proportion is,
As the Radius,
Is to the co-sine of the difference of Longit.
So is the co-tangent of one of the Latitudes
To the tangent of another ark.

Which being substracted out of the other Latitude, and 90 degrees added thereto, say;

As

As the co-fine of the ark found,
Is to the co-fine of the ark remaining;
So is the co-fine of the Latitude first taken,
To the co-fine of the distance.

Probl. 7.

The Rumbe and distance of two places given, to finde the difference of Latitude.

THe proportion is: As the Radius, Is to the co-fine of the rumb from the meridian: So is the distance, To the difference of Latitude.

Example.

If a ship fail West-north-west, (that is, upon the fixt rumb from the meridian) the distance of 90 leagues; what shall be the difference of Latitude?

First, I seek in the Table of Angles which every Rumb maketh with the Meridian, for the quantity of the angle of the fixt rumb, which is 67 degr. 50 parts, the complement whereof is 22 degr. 50 parts; therefore,

As the Radius,	10.000000
Is to the fine of, 22,50	9.582838
So is the distance in leagues 90,	1.954243
To the difference of Latitude 34, and better	1.537081

And

And by looking the next neerest Logarithm, the difference of latitude will be 34 leagues, and 44 hundred parts of a league.

And because 5 centesmes of a degree answereth to one league, therefore if you multiply 3444 by 5, the product will be 17220, from which cutting off the four last figures, the difference of latitude will be one degree 72 centesmes of a degree, and somewhat more.

Probl. 8.

The Rumb and Latitude of two places being given, to finde the difference of Longitude.

THe proportion is: As the Radius, Is to the tangent of the rumb from the meridian: So is the proper difference of latitude, To the difference of Longitude.

Example.

If a ship sail West-north-west (that is, upon the sixt Rumb from the meridian) so far, that from the latitude of 51 degrees, 53 centesmes, it cometh to the latitude of 49 degrees, 82 centesmes; what difference of Longitude hath such a course made?

First, I seek in the Table of Meridional parts what degrees do there answer to each latitude

latitude, and to 51 degrees, 53 min. I finde 60.328, and to 49 degrees, 82 minutes, 57.629, which being substracted from 60.328 their difference is 2.699, the proper difference of latitude. Therefore,

As the Radius,	10.000000
To the tangent of 67.50.	10.382775
So is 2.699.	0.431203

To 6 the difference of Longitude, 0.813978

Or in minuter parts 6.515, that is 6 degr. 51 centesmes of a degree *fere*, which was the thing required.

Here followeth the Table of Meridional parts, mentioned in some of the preceding Problemes, together with other Tables usefull in the Arts of Dialling and Navigation.

A Table of Meridional parts.

M.	Gr.par	M.	Gr.par	M.	G.par
0.00	0.000	3.00	3.001	6.00	6.011
0.10	0.100	3.10	3.101	6.10	6.111
0.20	0.200	3.20	3.201	6.20	6.212
0.30	0.300	3.30	3.301	6.30	6.312
0.40	0.400	3.40	3.402	6.40	6.413
0.50	0.500	3.50	3.502	6.50	6.514
0.60	0.600	3.60	3.602	6.60	6.614
0.70	0.700	3.70	3.702	6.70	6.715
0.80	0.800	3.80	3.803	6.80	6.816
0.90	0.900	3.90	3.903	6.90	6.916
1.00	1.000	4.00	4.003	7.00	7.017
1.10	1.100	4.10	4.103	7.10	7.118
1.20	1.200	4.20	4.204	7.20	7.219
1.30	1.300	4.30	4.304	7.30	7.319
1.40	1.400	4.40	4.404	7.40	7.420
1.50	1.500	4.50	4.504	7.50	7.521
1.60	1.600	4.60	4.605	7.60	7.622
1.70	1.700	4.70	4.705	7.70	7.723
1.80	1.800	4.80	4.805	7.80	7.824
1.90	1.900	4.90	4.906	7.90	7.925
2.00	2.000	5.00	5.006	8.00	8.026
2.10	2.100	5.10	5.106	8.10	8.127
2.20	2.200	5.10	5.207	8.20	8.228
2.30	2.300	5.30	5.307	8.30	8.329
2.40	2.400	5.40	5.408	8.40	8.430
2.50	2.500	5.50	5.508	8.50	8.531
2.61	2.600	5.60	5.609	8.60	8.632
2.71	2.700	5.70	5.709	8.70	8.733
2.81	2.800	5.80	5.810	8.80	8.834
2.91	2.900	5.90	5.910	8.90	8.936
3.01	3.000	6.00	6.011	9.00	9.037

A Table of Meridional parts.

M.	Gr.par	M.	Gr.par	M.	Gr.par
9.00	9.037	12.00	12.088	15.00	15.174
9.10	9.138	12.10	12.190	15.10	15.277
9.20	9.239	12.20	12.293	15.20	15.381
9.30	9.341	12.30	12.395	15.30	15.48[illegible]
9.40	9.442	12.40	12.497	15.40	15.58[illegible]
9.50	9.543	12.50	12.600	15.50	15.692
9.60	9.645	12.60	12.702	15.60	15.796
9.70	9.746	12.70	12.805	15.70	15.900
9.80	9.848	12.80	12.907	15.80	16.004
9.90	9.949	12.90	13.010	15.90	16.107
10.00	10.051	13.00	13.112	16.00	16.211
10.10	10.152	13.10	13.216	16.10	16.316
10.20	10.254	13.20	13.318	16.20	16.42[illegible]
10.30	10.355	13.30	13.422	16.30	16.5[illegible]
10.40	10.457	13.40	13.523	16.40	16.628
10.50	10.559	13.50	13.626	16.50	16.732
10.60	10.661	13.60	13.729	16.60	16.836
10.70	10.762	13.70	13.832	16.70	16.941
10.80	10.864	13.80	13.935	16.80	17.045
10.90	10.966	13.90	14.038	16.90	17.150
11.00	11.068	14.00	14.141	17.00	17.255
11.10	11.170	14.10	14.244	17.10	17.359
11.20	11.272	14.20	14.347	17.20	17.464
11.30	11.374	14.30	14.450	17.30	17.56[illegible]
11.40	11.476	14.40	14.553	17.40	17.673
11.50	11.578	14.50	14.656	17.50	17.778
11.60	11.680	14.60	14.760	17.60	17.883
11.70	11.782	14.70	14.863	17.70	17.988
11.80	11.884	14.80	14.967	17.80	18.09[illegible]
11.90	11.986	14.90	15.070	17.90	18.198
12.00	12.088	15.00	15.174	18.00	18.30[illegible]

A Table of Meridional parts.

M.	Gr. par	M.	G. par	M.	Gr. par
18.00	18.303	21.00	21.486	24.00	24.734
18.10	18.408	21.10	21.593	24.10	24.844
18.20	18.513	21.20	21.701	24.20	24.953
18.30	18.619	21.30	21.808	24.30	25.063
18.40	18.724	21.40	21.915	24.40	25.173
18.50	18.830	21.50	21.023	24.50	25.282
18.60	18.935	21.60	22.130	24.60	25.392
18.70	19.041	21.70	22.238	24.70	25.502
18.80	19.146	21.80	22.345	24.80	25.613
18.90	19.251	21.90	22.453	24.90	25.723
19.00	19.356	22.00	22.561	25.00	25.833
19.10	19.463	22.10	22.669	25.10	25.943
19.20	19.569	22.20	22.777	25.20	26.054
19.30	19.675	22.30	22.885	25.30	26.164
19.40	19.781	22.40	22.993	25.40	26.275
19.50	19.887	22.50	23.101	25.50	26.386
19.60	19.993	22.60	23.210	25.60	26.497
19.70	20.100	22.70	23.318	25.70	26.608
19.80	20.206	22.80	23.427	25.80	26.719
19.90	20.312	22.90	23.535	25.90	26.830
20.00	20.419	23.00	23.643	26.00	26.941
20.10	20.525	23.10	23.752	26.10	27.052
20.20	20.632	23.20	23.861	26.20	27.164
20.30	20.738	23.30	23.970	26.30	27.275
20.40	20.845	23.40	24.079	26.40	27.387
20.50	20.952	23.50	24.188	26.50	27.499
20.60	21.059	23.60	24.297	26.60	27.610
20.70	21.165	23.70	24.406	26.70	27.722
20.80	21.272	23.80	24.515	26.80	27.834
20.90	21.379	23.90	24.624	26.90	27.946
21.00	21.486	24.00	24.734	27.00	28.058

A Table of Meridional parts.

M.	Gr.par	M.	Gr.par	M.	Gr.par
27.00	28.058	30.00	31.473	33.00	34.992
27.10	28.171	30.10	31.588	33.10	35.111
27.20	28.283	30.20	31.704	33.20	35.231
27.30	28.396	30.30	31.820	33.30	35.350
27.40	28.508	30.40	31.936	33.40	35.470
27.50	28.621	30.50	32.052	33.50	35.590
27.60	28.734	30.60	32.168	33.60	35.710
27.70	28.847	30.70	32.284	33.70	35.830
27.80	28.959	30.80	32.409	33.80	35.950
27.90	29.072	30.90	32.517	33.90	36.071
28.00	29.186	31.00	32.633	34.00	36.191
28.10	29.299	31.10	32.750	34.10	36.312
28.20	29.413	31.20	32.867	34.20	36.433
28.30	29.526	31.30	32.984	34.30	36.554
28.40	29.640	31.40	33.101	34.40	36.675
28.50	29.753	31.50	33.218	34.50	36.796
28.60	29.867	31.60	33.336	34.60	36.917
28.77	29.981	31.70	33.453	34.70	37.039
28.80	30.095	31.80	33.571	34.80	37.161
28.90	30.300	31.90	33.688	34.90	37.283
29.00	30.324	32.00	33.806	35.00	37.405
29.10	30.438	32.10	33.924	35.10	37.527
29.20	30.553	32.20	34.042	35.20	37.649
29.30	30.667	32.30	34.161	35.30	37.771
29.40	30.782	32.40	34.279	35.40	37.894
29.50	30.897	32.50	34.397	35.50	38.017
29.60	31.012	32.60	34.516	35.60	38.140
29.70	31.127	32.70	34.635	35.70	38.263
29.80	31.242	32.80	34.754	35.80	38.386
29.90	31.357	32.90	34.873	35.90	38.509
30.00	31.473	33.00	34.992	36.00	38.633

A Table of Meridional parts.

M.	G. par	M.	G. par	M.	G. par
36.00	38.633	39.00	42.415	42.00	46.362
36.10	38.757	39.10	42.544	42.10	46.496
36.20	38.880	39.20	42.673	42.20	46.631
36.30	39.004	39.30	42.802	42.30	46.766
36.40	39.129	39.40	42.931	42.40	46.902
36.50	39.253	39.50	43.061	42.50	47.037
36.60	39.377	39.60	43.191	42.60	47.173
36.70	39.502	39.70	43.320	42.70	47.309
36.80	39.627	39.80	43.451	42.80	47.445
36.90	39.752	39.90	43.581	42.90	47.581
37.00	39.877	40.00	43.711	43.00	47.718
37.10	40.002	40.10	43.842	43.10	47.855
37.20	40.128	40.20	43.973	43.20	47.992
37.30	40.253	40.30	44.104	43.30	48.129
37.40	40.379	40.40	44.235	43.40	48.267
37.50	40.505	40.50	44.366	43.50	48.404
37.60	40.631	40.60	44.498	43.60	48.542
37.70	40.757	40.70	44.630	43.70	48.681
37.80	40.884	40.80	44.762	43.80	48.819
37.90	41.011	40.90	44.894	43.90	48.958
38.00	41.137	41.00	45.026	44.00	49.097
38.10	41.264	41.10	45.159	44.10	49.236
38.20	41.392	41.20	45.292	44.20	49.375
38.30	41.519	41.30	45.425	44.30	49.515
38.40	41.646	41.40	45.558	44.40	49.655
38.50	41.774	41.50	45.691	44.50	49.795
38.60	41.902	41.60	45.825	44.60	49.935
38.70	42.030	41.70	45.959	44.70	50.076
38.80	42.158	41.80	46.093	44.80	50.217
38.90	42.287	41.90	46.227	44.90	50.358
39.00	42.415	42.00	46.362	45.00	50.499

A Table of Meridional parts.

M.	G. par.	M.	G. par.	M.	G. par.
45.00	50.499	48.00	54.860	51.00	59.481
45.10	50.64[illegible]	48.10	55.010	51.10	59.640
45.20	50.783	48.20	55.160	51.20	59.800
45.30	50.9[illegible]5	48.30	55.31[illegible]	51.30	59.960
45.40	51.068	48.40	55.460	51.40	60.120
45.50	51.210	48.50	55.611	51.50	60.280
45.60	51.353	48.60	55.762	51.60	60.441
45.70	51.496	48.70	55.913	51.70	60.601
45.80	51.639	48.80	56.065	51.80	60.763
45.90	51.783	48.90	56.217	51.90	60.925
46.00	51.927	49.00	56.369	52.00	61.088
46.10	52.071	49.10	56.522	52.10	61.250
46.20	52.215	[illegible]	[illegible]	[illegible]	[illegible]
46.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
46.40	52.505	49.40	56.98[illegible]	[illegible]	61.740
46.50	52.650	49.50	57.135	52.50	61.904
46.60	52.795	49.60	57.289	52.60	62.068
46.70	52.941	49.70	57.444	52.70	62.23[illegible]
46.80	53.087	49.80	57.598	52.80	62.39[illegible]
46.90	53.233	49.90	57.754	52.90	62.56[illegible]
47.00	53.380	50.00	57.909	53.00	62.73[illegible]
47.10	53.526	50.10	58.065	53.10	62.8[illegible]
47.20	53.673	50.20	58.221	53.20	63.0[illegible]
47.30	53.821	50.30	58.377	53.30	63.2[illegible]
47.40	53.968	50.40	58.534	53.40	63.3[illegible]
47.50	54.116	50.50	58.691	53.50	63.56[illegible]
47.60	54.264	50.60	58.848	53.60	63.73[illegible]
47.70	54.413	50.70	59.006	53.70	63.9[illegible]
47.80	54.562	50.80	59.164	53.80	64.07[illegible]
47.90	54.711	50.90	59.322	53.90	64.24[illegible]
48.00	54.860	51.00	59.481	54.00	64.41[illegible]

A Table of Meridionall parts.

M.	*G.par*	*M.*	*G.par*	*M.*	*G.par*
54.00	64.412	57.00	69.711	60.00	75.456
54.10	64.582	57.10	69.895	60.10	75.650
54.20	64.753	57.20	70.080	60.20	75.857
54.30	64.924	57.30	70.263	60.30	76.059
54.40	65.096	57.40	70.449	60.40	76.261
54.50	65.268	57.50	70.635	60.50	76.464
54.60	65.440	57.60	70.821	60.60	76.667
54.70	65.613	57.70	71.008	60.70	76.871
54.80	65.786	57.80	71.195	60.80	77.076
54.90	65.960	57.90	71.383	60.90	77 281
55.00	66.134	58.00	71.572	61.00	77.487
55.10	66.308	58.10	71.761	61.10	77.694
55.20	66.483	58.20	71.950	61.20	77.901
55.30	66.659	58.30	72.140	61.30	78.109
55.40	66.835	58.40	72.331	61.40	78.317
55.50	67.011	58.50	72.522	61.50	78.526
55.60	67.188	58.60	72.714	61.60	78.736
55.70	67.365	58.70	72.906	61.70	78.947
55.80	67.543	58.80	73.099	61.80	79.158
55.90	67.721	58.90	73.292	61.90	79.370
56.00	67.900	59.00	73.486	62.00	79.583
56.10	68.079	59.10	73.680	62.10	79.796
56.20	68.258	59.20	73.875	62.20	89.010
56.30	68.438	59.30	74.071	62.30	89.225
56.40	68.616	59.40	74.267	62.40	89.441
56.50	68.799	59.50	74.464	62.50	89.657
56.60	68.981	59.60	74.661	62.60	89.874
56.70	69.163	59.70	74.859	62.70	81.091
56.80	69.345	59.80	75.057	62.80	81.310
56.90	69.528	59.90	75.256	62.90	81.529
57.00	69.711	60.00	75.456	63.00	81.749

A Table of Meridional parts.

M.	G.par.	M.	G.par.	M.	G.par.
63.00	81.749	66.00	88.725	69.00	96.575
63.10	81.970	66.10	88.971	69.10	96.854
63.20	82.191	66.20	89.219	69.20	97.135
63.30	82.413	66.30	89.467	69.30	97.418
63.40	82.635	66.40	89.716	69.40	97.701
63.50	82.860	66.50	89.967	69.50	97.986
63.60	83.084	66.60	90.218	69.60	98.272
63.70	83.310	66.70	90.470	69.70	98.560
63.80	83.536	66.80	90.723	69.80	98.849
63.90	83.763	66.90	90.978	69.90	99.139
64.00	83.990	67.00	91.232	70.00	99.431
64.10	84.219	67.10	91.489	70.10	99.724
64.20	84.448	67.20	91.746	70.20	100.018
64.30	84.678	67.30	92.005	70.30	100.314
64.40	84.909	67.40	92.264	70.40	100.612
64.50	85.141	67.50	92.525	70.50	100.910
64.60	85.374	67.60	92.787	70.60	101.211
64.70	85.607	67.70	93.050	70.70	101.513
64.80	85.842	67.80	93.314	70.80	101.816
64.90	86.077	67.90	93.579	70.90	102.121
65.00	86.313	68.00	93.846	71.00	102.427
65.10	86.550	68.10	94.113	71.10	102.735
65.20	86.788	68.20	94.382	71.20	103.044
65.30	87.027	68.30	94.652	71.30	103.356
65.40	87.267	68.40	94.923	71.40	103.664
65.50	87.508	68.50	95.195	71.50	103.983
65.60	87.749	68.60	95.468	71.60	104.299
65.70	87.992	68.70	95.743	71.70	104.616
65.80	88.235	68.80	96.019	71.80	104.936
65.90	88.480	68.90	96.296	71.90	105.259
66.00	88.725	69.00	96.575	72.00	105.579

A Table of Meridional parts.

M.	Gr.par	M.	Gr.par	M.	Gr.par
72.00	105 579	75.00	116 171	78.00	129 075
72.10	105 904	75.10	116 559	78.10	129 558
72.20	106 230	75.20	116 949	78.20	130 045
72.30	106 558	75.30	117 342	78.30	130 536
72.40	106 888	75.40	117 737	78.40	131 031
72.50	107 220	75.50	118 135	78.50	131 530
72.60	107 553	75.60	118 536	78.60	132 034
72.70	107 888	75.70	118 939	78.70	132 542
72.80	108 226	75.80	119 345	78.80	133 055
72.90	108 565	75.90	119 755	78.90	133 572
73.00	108 906	76.00	120 165	79.00	134 094
73.10	109 249	76.10	120 581	79.10	134 620
73.20	109 594	76.20	121 000	79.20	135 151
[illegible]	[illegible]	76.30	121 420	79.30	135 687
73.40	[illegible]	[illegible]	[illegible]	79.40	136 [illegible]
73.50	110 641	76.50	122 270	79.50	136 775
73.60	110 994	76.60	122 700	79.60	137 326
73.70	111 349	76.70	123 133	79.70	137 883
73.80	111 707	76.80	123 570	79.80	138 445
73.90	112 066	76.90	124 009	79.90	139 012
74.00	112 428	77.00	124 452	80.00	139 585
74.10	112 792	77.10	124 898	80.10	140 164
74.20	113 158	77.20	125 348	80.20	140 748
74.30	113 526	77.30	125 801	80.30	141 339
74.40	113 897	77.40	126 258	80.40	141 936
74.50	114 270	77.50	126 718	80.50	142 138
74.60	114 645	77.60	127 182	80.60	143 147
74.70	115 023	77.70	127 649	80.70	143 763
74.80	115 403	77.80	128 121	80.80	144 385
74.90	115 786	77.90	128 596	80.90	145 014
75.00	116 171	78.00	129 075	81.00	145 650

A Table of Meridional parts.

M.	Gr. par.	M.	Gr. par.	M.	Gr. par.
81 00	145 650	84 00	168 947	87 00	208 7[illegible]
81 10	146 292	84 10	169 912	87 10	210 649
81 20	146 942	84 20	170 893	87 20	212 668
81 30	147 600	84 30	171 891	87 30	214 745
81 40	148 265	84 40	172 907	87 40	216 909
81 50	148 937	84 50	173 941	87 50	219 158
81 60	149 618	84 60	174 994	87 60	221 498
81 70	150 307	84 70	176 067	87 70	223 938
81 80	151 003	84 80	177 160	87 80	226 486
81 90	151 709	84 90	178 275	87 90	229 153
82 00	152 423	85 00	179 411	88 00	231 95[illegible]
82 10	153 147	85 10	180 569	88 10	234 891
82 20	153 878	85 20	181 752	88 20	237 991
82 30	154 620	85 30	182 960	88 30	241 [illegible]
82 40	155 372	85 40	184 194	88 40	244 7[illegible]
82 50	156 132	85 50	185 454	88 50	248 445
82 60	156 903	85 60	186 743	88 60	252 4[illegible]
82 70	157 685	85 70	188 062	88 70	256 [illegible]
82 80	158 478	85 80	189 411	88 80	261 243
82 90	159 281	85 90	190 793	88 90	266 235
83 00	160 096	86 00	192 210	89 00	271 701
83 10	160 922	86 10	193 661	89 10	276 753
83 20	161 761	86 20	195 151	89 20	284 517
83 30	162 612	86 30	196 680	89 30	292 191
83 40	163 475	86 40	198 251	89 40	303 958
83 50	164 352	86 50	199 867	89 50	311 [illegible]
83 60	165 242	86 60	201 529	89 60	324 465
83 70	166 146	86 70	203 240	89 70	341 166
83 80	167 065	86 80	205 005	89 80	365 [illegible]
83 90	167 999	86 90	206 825	89 90	408 [illegible]
84 00	168 947	87 00	208 705	90 00	Infinite

A Table of the Suns De-

1654, 1658,

Days	Ianu.	Febr.	Mar.	Apr.	May.	Iune
	ſouth	ſouth	[illegible]	north	north	north
1	21 98	13 85	3 48	08 52	18 03	23 18
2	21 62	13 52	3 10	08 88	18 28	23 25
3	21 45	13 17	2 70	09 25	18 53	23 30
4	21 27	12 83	2 30	09 60	18 77	23 35
5	21 08	12 50	1 92	09 97	19 02	23 40
6	20 88	12 15	1 52	10 31	19 23	23 43
7	20 68	11 80	1 11	10 67	19 47	23 [illegible]
8	20 48	11 43	0 72	11 02	19 68	23 50
9	20 27	11 08	0 33	11 36	19 90	23 [illegible]
10	10 05	10 72	0 06	11 70	20 11	23 52
11	19 82	10 37	N 47	12 05	20 31	23 53
12	[illegible]	09 83	0 85	12 38	20 51	23 [illegible]
13	19 35	09 63	1 25	12 72	20 70	23 [illegible]
14	19 11	09 25	1 65	13 05	20 88	23 [illegible]
15	18 86	08 88	2 03	13 36	21 06	23 46
16	18 61	08 52	2 41	13 68	21 25	23 43
17	18 35	08 13	2 82	14 00	21 41	23 40
18	18 08	07 75	3 20	14 31	21 58	23 35
19	17 [illegible]	07 37	3 60	14 63	21 73	23 30
20	17 53	06 98	3 98	14 93	21 88	23 25
21	17 25	06 60	4 37	15 23	22 03	23 18
22	16 96	06 22	4 75	15 53	22 16	23 10
23	16 68	05 83	5 13	15 83	22 30	23 03
24	16 38	05 45	5 51	16 13	22 41	22 95
25	16 08	05 07	5 90	16 41	22 53	22 [illegible]
26	15 78	04 67	6 28	16 70	22 65	22 75
27	15 46	04 28	6 67	16 97	22 75	22 65
28	15 15	03 88	7 03	17 23	22 85	22 53
29	14 83		7 41	17 50	22 95	22 41
30	14 51		7 78	17 77	23 03	22 30
31	14 18		8 15		23 77	

clination, for the years A

1662, 1666.

Dayes	July. north		Aug. north		Sep. nort		Octo. south		Nov. south		Dec. south	
1	22	16	15	28	4	50	7	15	17	63	23	13
2	22	03	14	98	4	11	7	53	17	86	23	20
3	21	89	14	66	3	73	7	91	18	13	23	26
4	21	73	14	36	3	35	8	28	18	40	23	32
5	21	58	14	05	2	96	8	65	18	66	23	38
6	21	42	13	73	3	56	9	03	18	91	23	43
7	21	25	13	41	2	18	9	40	19	15	23	46
8	21	07	13	08	1	80	9	76	19	40	23	48
9	20	90	12	76	1	40	10	13	19	63	23	50
10	20	71	12	43	1	01	10	48	19	86	23	51
11	20	51	12	10	0	63	10	85	20	08	23	53
12	20	31	11	76	0	23	11	20	20	30	23	51
13	20	11	11	43	0S	16	11	57	20	51	[illegible]	[illegible]
14	19	90	11	08	0	55	11	91	20	71	[illegible]	[illegible]
15	19	68	10	73	0	95	12	25	20	91	[illegible]	[illegible]
16	19	47	10	38	1	33	12	60	21	10	[illegible]	[illegible]
17	19	25	10	03	1	73	12	95	21	28	[illegible]	[illegible]
18	19	01	09	68	2	11	13	28	21	46	[illegible]	[illegible]
19	18	78	09	33	2	51	13	61	21	63	[illegible]	[illegible]
20	18	55	08	96	2	90	13	95	21	80	[illegible]	[illegible]
21	18	30	08	60	3	30	14	26	21	95	23	05
22	18	05	08	25	3	68	14	60	22	10	22	96
23	17	78	07	88	4	06	14	91	22	25	22	86
24	17	53	07	51	4	46	15	23	22	38	22	76
25	17	26	07	15	4	85	15	55	22	51	22	65
26	17	00	06	76	5	23	15	85	22	63	[illegible]	[illegible]
27	16	71	06	40	5	63	16	15	22	73	[illegible]	[illegible]
28	16	43	06	01	6	00	16	45	22	85	[illegible]	[illegible]
29	16	15	05	[illegible]	6	38	16	75	22	95	[illegible]	[illegible]
30	15	86	05	26	6	76	17	03	23	03	[illegible]	[illegible]
31	15	56	04	88			17	31			[illegible]	[illegible]

A Table of the Suns De-

1655, 1659.

Days	Janu. south		Febr. south		Mar. south		Apr. north		May. north		June north	
1	21	[illegible]	13	93	3	58	08	[illegible]	17	96	23	16
2	21	65	13	60	3	18	08	80	18	21	23	23
3	21	48	13	26	2	80	09	15	18	46	23	30
4	21	30	12	91	2	40	09	51	18	71	23	35
5	21	11	12	58	2	00	09	88	18	95	23	40
6	20	[illegible]	12	23	1	61	10	23	19	18	23	43
7	20	[illegible]	11	88	1	21	10	58	19	41	23	46
8	20	[illegible]	11	53	0	81	10	93	19	63	23	48
9	20	32	11	16	0	43	11	28	19	85	23	50
10	20	10	10	81	N	03	11	61	20	06	23	51
11	19	88	10	45	0	36	11	96	20	26	23	53
12	19	65	10	08	0	76	12	30	20	46	23	51
13	19	41	09	71	1	15	12	63	20	66	23	51
14	19	16	09	35	1	55	12	96	20	85	23	50
15	18	92	08	96	1	93	13	28	21	03	23	48
16	18	67	08	60	2	33	13	61	21	20	23	45
17	18	41	08	23	2	71	13	93	21	38	23	41
18	18	15	07	85	3	11	14	25	21	55	23	36
19	17	88	77	46	3	50	14	56	21	70	23	31
20	17	60	07	08	3	88	14	86	21	85	23	26
21	17	31	06	70	4	28	15	16	22	00	23	20
22	17	03	06	31	4	66	15	46	22	13	23	13
23	16	73	05	93	5	05	15	76	22	26	23	05
24	16	[illegible]	05	53	5	43	16	05	22	40	22	96
25	16	[illegible]	05	15	5	81	16	35	22	51	22	88
26	15	[illegible]	04	76	6	20	16	63	22	63	22	78
27	15	[illegible]	04	36	6	56	16	90	22	73	22	68
28	15	23	03	98	6	95	17	18	22	83	22	56
29	14	91			7	31	17	45	22	93	22	45
30	14	58			7	70	17	71	23	01	22	33
31	14	26			8	06			23	10		

clination, for the years 1663, 1667.

Days	July. north		Aug. north		Sep. nort		Octo. south		Nov. south		Dec. south	
1	22	23	15	35	4	58	7	06	17	53	23	10
2	22	06	15	05	4	20	7	43	17	80	23	12
3	21	91	14	75	3	81	7	81	18	06	23	25
4	21	76	14	43	3	43	8	20	18	33	23	31
5	21	61	14	13	3	05	8	56	18	60	23	36
6	21	45	13	82	2	66	8	93	18	85	23	41
7	21	28	13	50	2	28	9	30	19	10	23	45
8	21	11	13	16	1	93	9	66	19	33	23	48
9	20	93	12	85	1	50	10	03	19	56	23	50
10	20	75	12	51	1	10	10	40	19	80	23	[illegible]
11	20	56	12	18	0	71	10	76	20	03	23	[illegible]
12	20	36	11	84	0	33	11	11	20	25	[illegible]	[illegible]
13	20	16	11	51	0	06	11	46	20	46	[illegible]	[illegible]
14	19	95	11	16	0	46	11	81	20	66	23	50
15	19	75	10	81	0	85	12	16	20	86	23	46
16	19	53	10	65	1	25	12	51	21	06	23	43
17	19	30	10	13	1	63	12	86	21	25	23	40
18	19	06	09	76	2	03	13	20	21	41	23	35
19	18	83	09	41	2	41	13	53	21	60	23	28
20	18	60	09	06	2	80	13	86	21	76	23	21
21	18	35	08	70	3	20	14	20	21	91	23	15
22	18	10	08	33	3	60	14	51	22	06	23	06
23	17	85	07	96	3	98	14	83	22	21	22	[illegible]
24	17	58	07	60	4	36	15	15	22	35	22	[illegible]
25	17	31	07	23	4	76	15	46	22	48	22	[illegible]
26	17	05	06	85	5	15	15	76	22	60	22	[illegible]
27	16	78	06	48	5	53	16	08	22	71	22	[illegible]
28	16	50	06	11	5	91	16	38	22	83	22	[illegible]
29	16	21	05	73	6	30	16	68	22	93	22	[illegible]
30	15	93	05	36	6	68	16	96	23	01	22	[illegible]
31	15	65	04	98			17	25			22	01

A Table of the Suns De-

1656. 1660.

Dayes	Ianu. south		Febr. south		Mar south		Apr. north		May. north		June north	
1	21	85	14	01	3	28	08	70	18	16	23	21
2	21	70	13	68	3	90	09	06	18	41	23	28
3	21	53	13	35	2	50	09	43	18	65	23	33
4	21	35	13	00	2	10	09	78	18	90	23	38
5	21	16	12	66	1	66	10	15	19	13	23	41
6	20	98	12	31	1	33	10	50	19	35	23	45
7	20	78	11	96	0	91	10	85	19	58	23	48
8	20	58	11	61	0	53	11	20	19	80	23	50
9	20	36	11	26	0	13	11	53	20	01	23	51
10	20	15	10	90	N	26	11	88	20	21	23	52
11	19	93	10	53	0	66	12	21	20	41	23	53
12	19	70	10	16	1	05	12	55	20	61	23	51
13	19	46	09	80	1	45	12	88	20	80	23	50
14	19	23	09	43	1	83	13	20	20	98	23	48
15	18	98	09	06	2	23	13	53	21	16	23	45
16	18	73	08	70	2	63	13	85	21	33	23	41
17	18	48	08	33	3	03	14	16	21	50	23	38
18	18	21	07	93	3	41	14	48	21	66	23	33
19	17	95	07	55	3	80	14	80	21	81	23	26
20	17	66	07	16	4	18	15	10	21	96	23	21
21	17	40	06	78	4	56	15	40	22	10	23	15
22	17	11	06	40	4	95	15	70	22	23	23	06
23	16	81	06	01	5	33	15	98	22	36	22	98
24	16	53	05	63	5	71	16	28	22	48	22	90
25	16	23	05	25	6	10	16	56	22	60	22	80
26	15	91	04	86	6	48	16	83	22	71	22	70
27	15	61	04	48	6	85	17	11	22	81	22	60
28	15	30	04	06	7	23	17	38	22	90	22	48
29	14	98	03	68	7	60	17	65	22	98	22	35
30	14	66			7	96	17	90	23	06	22	21
31	14	35			8	33			23	15		

clination, for the years 1564, 1668.

Days	July north		Aug. north		Sep. nort		Octo. south		Nov. south		Dece. south	
1	22	04	15	11	4	3	7	35	17	73	23	16
2	21	95	14	81	3	91	7	73	18	00	23	23
3	21	80	14	51	3	53	8	10	18	26	23	30
4	21	65	14	20	3	15	8	48	18	53	23	35
5	21	50	13	88	2	75	8	85	18	79	23	40
6	21	33	13	56	2	36	9	21	19	03	23	45
7	21	16	13	25	1	98	9	58	19	28	23	48
8	20	98	12	91	1	[illegible]	9	95	19	51	23	50
9	20	80	12	63	1	20	10	31	19	75	23	51
10	20	61	12	26	0	81	10	68	19	93	23	52
11	20	41	11	93	0	41	11	03	20	23	23	53
12	20	21	11	60	0	03	11	38	20	41	23	51
13	20	00	11	25	0	36	11	73	20	61	23	50
14	19	80	10	90	0	75	12	08	20	81	23	48
15	19	58	10	56	1	15	12	43	21	01	23	45
16	19	35	10	21	1	55	12	76	21	20	23	40
17	19	13	9	85	1	93	13	11	21	38	23	35
18	18	90	9	50	2	31	13	45	21	55	23	30
19	18	65	9	15	2	71	13	78	21	71	23	25
20	18	41	8	79	3	10	14	11	21	88	23	18
21	18	16	8	41	3	50	14	43	22	03	23	10
22	17	91	8	05	3	88	14	76	22	18	23	01
23	17	66	7	68	4	28	15	08	22	31	22	91
24	17	40	7	31	4	66	15	43	22	45	22	81
25	17	11	6	95	5	05	15	70	22	58	22	70
26	16	85	6	56	5	43	16	00	22	70	22	58
27	16	56	6	29	5	81	16	30	22	80	22	46
28	16	28	5	83	6	20	16	60	22	90	22	33
29	16	00	5	45	6	58	16	90	23	00	22	20
30	15	71	5	06	6	96	17	18	23	08	22	05
31	15	41	4	68			17	46			21	90

A Table of the Suns De-

1657. 1661.

Dayes.	Ianu. south	Febr. south	Mar sout	Apr. north	May. north	Iune north
1	21 73	13 76	3 40	08 60	18 08	23 2[illegible]
2	21 56	13 43	3 00	08 95	18 33	23 26
3	21 38	13 08	2 61	09 33	18 58	23 31
4	21 21	12 75	2 21	09 70	18 83	23 36
5	21 03	12 41	1 81	10 05	19 06	23 41
6	20 83	12 06	1 41	10 40	19 30	23 45
7	20 63	11 71	1 01	10 75	19 51	23 48
8	20 43	11 35	0 63	11 10	19 73	23 50
9	20 21	11 00	0 23	11 45	19 95	23 51
10	20 00	10 63	N 6	11 78	20 16	23 52
11	19 76	10 26	0 55	12 11	20 36	23 53
12	19 53	09 90	0 95	12 46	20 56	23 51
13	19 30	09 53	1 35	12 80	20 75	23 50
14	19 05	09 16	1 73	13 11	20 93	23 48
15	18 80	08 80	2 13	13 45	21 11	23 46
16	18 55	08 41	2 51	13 76	21 28	23 42
17	18 28	08 05	2 90	14 08	21 45	23 38
18	18 03	07 66	3 30	14 40	21 61	23 33
19	17 75	07 28	3 68	14 70	21 76	23 28
20	17 46	06 90	3 08	15 01	21 91	23 23
21	17 18	06 51	4 46	15 31	22 06	23 16
22	16 90	06 13	4 85	15 61	22 20	23 10
23	16 60	05 75	5 23	15 90	22 33	23 01
24	16 30	05 35	5 61	16 20	22 45	22 91
25	16 00	04 96	6 00	16 48	22 56	22 83
26	15 70	04 56	6 36	16 76	22 68	22 73
27	15 38	04 18	6 75	17 03	22 78	22 61
28	15 05	03 78	7 11	17 30	22 88	22 51
29	14 75		7 50	17 56	22 95	22 38
30	14 43		7 86	17 83	23 05	22 26
31	14 10		8 23		23 13	

clination, for the years 1665, 1669.

Dayes	July north		Aug. north		Sep. nort		Octo south		Nov south		Dece. south	
1	22	13	15	20	4	40	7	25	17	66	23	19
2	22	00	14	99	4	01	7	63	17	93	23	[illegible]
3	21	85	14	60	3	63	8	00	18	20	23	[illegible]
4	21	70	14	28	3	25	8	36	18	46	23	[illegible]
5	21	53	13	96	2	86	8	75	18	71	23	[illegible]
6	21	36	13	65	2	48	9	11	18	96	23	[illegible]
7	21	20	13	33	2	08	9	48	19	21	23	[illegible]
8	21	03	13	01	1	70	9	84	19	45	23	[illegible]
9	20	85	12	68	1	31	10	21	19	68	23	[illegible]
10	20	65	12	35	0	91	10	58	19	91	23	[illegible]
11	20	46	12	01	0	53	10	93	20	13	23	[illegible]
12	20	26	11	68	0	13	11	30	20	35	23	[illegible]
13	20	06	11	35	0S	26	11	65	20	56	23	50
14	19	85	11	15	0	65	12	00	20	76	23	[illegible]
15	19	63	10	65	1	05	12	35	20	96	23	[illegible]
16	19	41	10	30	1	43	12	68	21	15	23	[illegible]
17	19	20	9	95	1	83	13	02	21	33	23	[illegible]
18	18	96	9	63	2	21	13	36	21	51	23	[illegible]
19	18	71	9	25	2	61	13	70	21	68	23	[illegible]
20	18	48	8	88	3	00	14	03	21	83	23	20
21	18	23	8	51	3	38	14	35	22	00	23	[illegible]
22	17	98	8	15	3	78	14	68	22	15	23	[illegible]
23	17	73	7	78	4	16	15	00	22	28	22	95
24	17	46	7	41	4	55	15	32	22	41	22	[illegible]
25	17	20	7	05	4	95	15	61	22	55	22	[illegible]
26	16	93	6	68	5	33	15	91	22	66	22	[illegible]
27	16	65	6	30	5	71	16	21	22	76	22	[illegible]
28	16	36	5	93	6	10	16	51	22	86	22	[illegible]
29	16	08	5	55	6	48	16	81	22	96	22	[illegible]
30	15	80	5	16	6	86	17	10	23	06	22	[illegible]
31	15	50	4	78			17	38			21	93

A Table of the Suns right Ascension in hours and minutes.

Dayes	Janu. H.	M.	Febr. H.	M.	Mar. H.	M.	Apr. H	M	May H.	M.	June H.	M
1	19	53	21	68	23	45	1	33	3	11	5	30
2	19	61	21	75	23	51	1	39	3	18	5	36
3	19	68	21	81	23	56	1	45	3	35	5	43
4	19	75	21	88	23	63	1	51	3	40	5	55
5	19	83	21	95	23	70	1	56	3	46	5	56
6	19	90	22	00	23	75	1	63	3	53	5	63
7	19	96	22	06	23	81	1	70	3	65	5	71
8	20	05	22	13	23	86	1	75	3	66	5	78
9	20	11	22	20	23	93	1	81	3	73	5	85
10	20	18	22	26	00	00	1	88	3	80	5	91
11	20	25	22	33	0	05	1	95	3	86	5	98
12	20	33	22	40	0	11	2	00	3	93	6	05
13	20	40	22	45	0	18	2	06	4	00	6	13
14	20	46	22	51	0	23	2	13	4	06	6	20
15	20	53	22	58	0	30	2	18	4	13	6	26
16	20	60	22	65	0	35	2	25	4	20	6	33
17	20	66	22	70	0	41	2	31	4	26	6	40
18	20	75	22	76	0	48	2	38	4	33	6	46
19	20	81	22	83	0	53	2	45	4	40	6	53
20	20	88	22	90	0	60	2	50	4	46	6	61
21	20	95	22	95	0	66	2	56	4	55	6	68
22	21	[illegible]	23	01	0	71	2	63	4	61	6	75
23	21	08	23	08	0	78	2	70	4	68	6	81
24	21	15	23	13	0	83	2	76	4	75	6	88
25	21	21	23	20	0	90	2	81	4	81	6	95
26	21	28	23	26	0	96	2	89	4	88	7	01
27	21	35	23	33	1	01	2	95	4	95	7	10
28	21	41	23	38	1	08	3	01	5	01	7	16
29	21	48			1	15	3	08	5	08	7	23
30	21	55			1	20	3	15	5	16	7	30
31	21	61			1	26			5	23		

A Table of the Suns right Ascension in hours and minutes.

Dayes	Jul H	Jul M	Aug. H	Aug. M	Sept. H	Sept. M	Octo. H	Octo. M	Nov. H	Nov. M	Dece. H	Dece. M
1	7	36	9	40	11	30	13	10	15	10	17	23
2	7	43	9	46	11	35	13	16	15	16	17	31
3	7	50	9	51	11	41	13	23	15	23	17	38
4	7	56	9	58	11	48	13	28	15	30	17	45
5	7	63	9	65	11	53	13	35	15	38	17	53
6	7	70	9	71	11	60	13	41	15	45	17	60
7	7	76	9	76	11	65	13	48	15	51	17	68
8	7	83	9	[illegible]	11	71	13	53	15	58	17	75
9	7	90	9	90	11	78	13	60	15	65	17	83
10	7	96	9	96	11	83	13	66	15	71	17	90
11	8	03	10	01	11	93	13	73	15	80	17	98
12	8	10	10	08	11	95	13	80	15	86	18	05
13	8	16	10	15	12	01	13	85	15	93	18	11
14	8	23	10	20	12	06	13	91	16	00	18	20
15	8	30	10	26	12	13	13	98	16	08	18	26
16	8	36	10	33	12	20	14	05	16	10	18	35
17	8	43	10	38	12	25	14	11	16	21	18	41
18	8	50	10	45	12	31	14	18	16	28	18	50
19	8	56	10	51	12	38	14	23	16	36	18	56
20	8	63	10	56	12	43	14	30	16	43	18	65
21	8	70	10	63	12	50	14	36	16	50	18	71
22	8	76	10	70	12	55	14	43	16	58	18	80
23	8	81	10	75	12	61	14	50	16	65	18	86
24	8	88	10	81	12	68	14	56	16	71	18	93
25	8	95	10	88	12	73	14	63	16	80	19	01
26	9	01	10	93	12	80	14	70	16	86	19	08
27	9	08	11	00	12	86	14	76	16	93	19	16
28	9	15	11	05	12	91	14	83	17	01	19	[illegible]
29	9	21	11	11	12	98	14	90	17	08	19	30
30	9	26	11	18	13	05	14	96	17	16	19	38
31	9	33	11	23			15	03			19	45

Declination and Right

The names of the Stars.	Declination D.	Declination M.		Dist. from the pole D.	Dist. from the pole M.	Right Ascention H.	Right Ascention M
Brest of *Cassiopeia*	54	76	N	35	24	0	35
Pole-star	87	48	N	02	52	0	51
Girdle of *Andromeda*	33	83	N	56	17	0	83
Knees of *Cassiopeia*	58	41	N	31	59	1	05
Whales belly	12	00	[illegible]	78	00	1	58
South. foot of *Andr.*	40	65	N	49	35	1	70
Rams head	21	81	N	68	19	1	80
Head of *Medusa*	39	58	N	50	42	2	76
Perseus right side	48	55	N	41	45	2	98
Buls eye	15	75	N	74	25	4	26
The Goat	45	58	N	44	42	4	85
Orions left foot	08	63	S	81	37	4	96
Orions left shoulder	05	98	N	84	02	5	10
First in *Orions* girdle	00	58	S	89	42	5	25
Second in *Orions* gird.	01	45	S	88	55	5	31
Third in *Orions* girdle	02	15	S	87	85	5	38
Wagoners right shold.	44	86	N	45	14	5	70
Orions right shoulder	07	30	N	82	70	5	60
Bright foot of Twins	16	65	N	73	35	6	30
The great Dog	16	21	S	73	79	6	50
Upper head of Twins	32	50	N	57	50	7	20
The lesse Dog	06	10	N	83	90	7	36
Lower head of Twins	28	80	N	61	20	7	40

Bright,

Ascension of the Stars.

Brightest in *Hydra*	21 16	S	82 84	09 16
Lions heart	13 63	N	76 37	09 83
Lions back	22 06	N	65 94	11 50
Lions tail	16 50	N	73 50	11 51
Great Bears rump.	58 72	N	31 28	10 66
First in the great Bears tail next her rump	57 85	N	32 15	12 63
Virgines spike	9 32	S	80 68	13 11
Middlemost in the great Bears tail	56 75	N	33 25	13 16
In the end of her tail	51 08	N	38 92	13 56
Between *Bootes* thighs	21 03	N	68 97	14 00
South Ballance	14 55	S	75 45	14 53
North Ballance	08 05	S	81 95	14 96
Scorpions heart	25 58	S	64 42	16 73
Hercules head	14 85	N	75 15	16 98
Serpentaries head	12 86	N	77 14	17 31
Dragons head	51 60	N	38 40	17 80
Brightest in the Harp	38 50	N	51 50	18 48
Eagles heart	08 02	N	81 98	19 56
Swans tail	44 08	N	45 92	20 50
Pegasus mouth	08 32	N	81 68	21 45
Pegasus shoulder	17 38	N	76 62	23 92
The head of *Androm.*	27 22	N	62 78	23 85

Rules for finding of the Poles elevation by the meridian altitude of the Sun or stars, and by the Table of their Declinations aforegoing.

Case 1.

If the Sun or star be on the meridian to the southwards, and have south declination. Adde the suns declination to his meridian altitude, and taking that total from 90 degrees, what remaineth is the latitude of the place desired.

As the 7 of *February*, 1654, by the aforegoing Table, the suns decl. south, is 11 . 80

The suns meridian altitude	15 . 27
The sum or total is	27 . 07
Which substracted from	90 . 00
There remaine the North latitude	62 . 93

But when you have added the suns declination to his meridian altitude, if the total exceed 90: substract 90 degr. from it, and what remaineth is your latitude to the southwards.

As admit the suns declination to be southerly 11 . 80

And his meridian altitude	87 . 23
The sum or total is	99 . 03
From which substracting	90 . 00
There remains the latitude south.	09 . 03

Case

Case 2.

If the sun or star be on the meridian to the southwards, and have north declination.

Substract the suns declination from his meridian altitude, and that which remains substract from 90, and then the remainer is the poles elevation northerly.

Case 3.

If the sun or star be on the meridian to the northwards, and have north declination

Adde the suns declination to his meridian altitude, the total take from 90, and what remaineth is the poles elevation southerly.

But when you have added the suns declination to his meridian altitude, if it exceed 90, substract 90 from it, and what remaineth is your latitude northerly.

Case 4.

If the sun be to the northwards at noon, and declination south.

Substract the suns declination from his meridian altitude, and that which remains substract from 90, what then remaineth is your latitude southerly.

And what is said of the Sun, is also to be understood of the Stars, being upon the Meridian.

Case

Case 5.

If you observe when the Sun hath no declination,

Substract his meridian altitude from 90, what remaineth is your latitude.

Case 6.

If you chance to observe when the Sun or star is in the Zenith, that is 90 degrees above the Horizon. Look in the table for the declination of the Sun or of that star, and the same is your latitude.

Case 7.

If the Sun come to the meridian under the Pole.

If you be within the Artick or Antartick circles, and observe the Sun upon the meridian under the Pole, substract the Suns declination from 90, the remainer is the Suns distance from the Pole; which distance added to his meridian altitude, the sum or total is the latitude sought.

And what is here said of the Sun is to be understood of the stars, whose declinations, distances from the pole, and right ascensions we have expressed in the foregoing Table.

FINIS.